ARITHMETIC REASONING

For most people, the math sections of Examinations are the most difficult. This section is one of the least popular, as it consists solely of mathematical word problems. Yet we've found that people can drastically improve their scores by practicing with word problems before the exam, and consciously cultivating good problem solving habits.

There are usually fifteen arithmetic reasoning questions. Most are of moderate difficulty, a few are pretty easy and two or three are sometimes very tricky. If you get stuck on one of the more difficult questions, you can put a check mark next to it and come back to it later if there's time. On most exams people don't usually run out of time, just patience. This is understandable, but points given for correctly answering one or two of these difficult questions may sometimes make a difference in ones place on a promotion list, so it's good to go back and try different approaches to solving a problem, rather than just guessing.

We suggest you check the answer key after every three questions. (Be sure to spend enough time on each question. Don't just give up quickly and turn to the answer key; (you won't get as much out of the process if you do). If you've missed any questions, consult the Self Study Guide and go through the explanation thoroughly before you continue on to the next question. (Don't worry if you don't do well at first, it's been a long time since most people have answered these types of questions). If your exam won't allow you to use a calculator, it's a good idea to practice these questions without a calculator to increase speed, accuracy and confidence. Remember, these questions are not easy for most people. They may seem really difficult at first and some may seem impossible. If, however, a real effort is made to practice and learn from mistakes, scores in this area can improve considerably. We suggest you do these problems again a week before the exam.

Good luck!

ARITHMETIC REASONING

1. If Jean's weekly income doubled she would be making $120 a week more than Barbara. Jean's weekly income is $80 more than half of Betty's. Betty makes $200 a week.

   How much does Barbara make?

   a. $180 c. $240
   b. $200 d. $360

2. A conference with 3600 participants gathers in Albany. One of every twelve people attending the conference who have ordered meals has special dietary needs. Half of those

attending the conference signed up for meals. How many have special dietary needs?

a. 266 c. 150
b. 133 d. 300

3. It costs $360 for Office X's service contract with a typewriter company to service 18 typewriters for six months. At this same rate, how much would it cost Office X to service six typewriters for three months?

a. $80 c. $90
b. $75 d. $60

4. In December, an office spent $480, or 15% of its non-personnel expenses that month, for postage. What were its total non-personnel expenses for December?

a. $3200 c. $5520
b. $552 d. $7200

5. Catherine bought an equal number of $11.00, $9.00, and $8.00 tickets for a concert. She spent $196 for all of the tickets. How many of each did she buy?

a. 6 c. 8
b. 7 d. Cannot be determined from information given

6. Agency Y employs 13,800 people. Of these, 42% are male, and 50% of the males are age 30 or younger. How many males are there in Agency Y who are older than 30?

a. 5796 c. 3471
b. 2898 d. 2910

7. A machine can collate 126 books, each with 400 page, in 14 days. If it continues to collate at this same rate, how many 400 page books could it collate in 30 days?

a. 256 c. 248
b. 290 d. 270

8. A typewriter and a dictation machine cost a total of $840. If the typewriter cost $360 more than the dictation machine, how much did the dictation machine cost?

a. $480 c. $240

b. \$440 d. \$280

9. A cabinet maker has a round piece of wood 1/2" in diameter and 3/4 yards long. She needs half the length for the back of a chair and the remaining piece for 3/4" pegs. How many pegs will she have?

a. 18 c. 10
b. 9 d. 8/9

10. Robin can wallpaper a room in four hours. Susan can wallpaper the same room in seven hours. How long will it take them to wallpaper the room if they work together?

a. 4.5 hours c. 5.5 hours
b. 3.5 hours d. 2.5 hours

11. Mary and Alice jog 3 miles each evening. If they run at a constant rate and it takes Mary 40 minutes while Alice finishes in half an hour, how much distance does Mary have left when Alice finishes?

a. 1 mile c. 2/3 mile
b. 3/4 mile d. 1.33 miles

12. As a fund raiser , a community organization buys tickets to the theater to resell 25% above cost. They buy 50 eight dollar tickets, 25 ten dollar tickets and 25 fifteen dollar tickets. If they sell all but three of the ten dollar tickets, how much money have they made?

a. \$218.75 c. \$248.75
b. \$1273 d. \$1243.75

13. If a couch cost \$640 after a 20% discount, what was its original price?

a. \$769 c. \$800
b. \$512 d. \$780

14. A salesperson traveled 145 miles Monday, 72 miles Tuesday, and 98 miles Wednesday for \$2300 worth of sales. If the business pays 21¢ a mile for gas and vehicle maintenance, approximately what percent of sales for the three days went to gas and vehicle maintenance?

a. 3%　　c. 6%
b. 8%　　d. 1%

15. If one of every eight junior year students at a high school takes Latin, approximately what percent of a junior year class of 650 took Latin?

a. 6　　c. 81
b. 14　　d. 13

16. The proposed budget for a new social service program is $102,000. The budget states that the project has secured $14,500 worth of transportation services and $1,200 worth of office equipment as in kind contributions. What percent of the budget has been secured in kind?

a. 21.4　　c. 20.9
b. 15.4　　d. 13.1

17. The purpose of the program in Question 16 is to distribute 250,000 pounds of food to disadvantaged persons. Approximately how many pounds of food will be distributed for each dollar budgeted for the program?

a. 3.73　　c. 2.45
b. 2.15　　d. 2.95

18. A community cannery charges 15¢ per quart and 25¢ for 2 pints for processing. People using the cannery can purchase jars there at 15% discount off the regular price of $4.25 per case of a dozen quart or pint jars. How much will it cost to can 76 quarts of tomatoes and 20 pints of jelly if the jars are bought at the cannery? (Jars are not sold individually).

a. $46.39　　c. $32.49
b. $37.14　　d. $13.90

19. A pharmacist combines ingredients x, y and z in a ratio of 1:2:7 to produce cough medicine. How many ounces of the second ingredient, ingredient y, is needed to make a 12 ounce bottle of the medicine?

a. 2.4　　c. 1.2
b. 8.4　　d. 3.6

20. Agency Y served 187,565 people in 1981. If the agency served 210,515 people in 1982, this reflected an increase of:

a. 19.10% c. 12.2%
b. 15.6% d. 10.9%

21. The number of people attending a weekly training program in the month of January averaged 116 people. If there were 105 people attending the first week, 106 the second, and 125 the third, how many people attended the fourth week?

a. 118 c. 130
b. 128 d. 124

22. It takes 16 typists 11 days to complete a project. How long would it take 10 typists, if they worked at the same rate to complete the same project?

a. 17.6 days c. 6.9 days
b. 6.8 days d. 18.4 days

23. If the sum of two numbers is 280, and their ratio is 7:3. then the smaller number is

a. 28 c. 56
b. 84 d. 196

24. The population of Metropolis county in 1982 is 130% of its population in 1972. The population in 1972 was 145,000. What was the population in 1982?

a. 196,425 c. 111,539
b. 174,612 d. 188,500

25. A car travels 50 miles an hour, and a plane travels 10 miles a minute. How far will the car travel when the plane travels 500 miles?

a. 50.4 miles c. 41.6 miles
b. 37.5 miles d. 39.7 miles

26. In a university with 2000 students the student-faculty ratio is 16:1. If 18% of the faculty have completed some of their own study at the university, approximately how many have not?

a. 119 c. 23
b. 127 d. 103

27. A discount house advertises that they sell all merchandise at cost plus 10%. If Jane buys a TV set for $300, approximately what is the stores profit?

a. $30.00 C. $27.27
b. $27.00 d. $32.26

28. From 6 p.m. until midnight, the temperature dropped at a constant rate. From midnight until 1 a.m., it dropped 8°. If at 6 p.m., the temperature was 54° and by 1 a.m., it was 37°, what was the temperature at 10 p.m.?

a. 46° c. 45°
b. 48° d. 49°

29. One eighth of a half gallon carton of ice cream has been eaten. The remainder is divided among three people. Approximately what percentage of a gallon does each person get?

a. 14.6% c. 29.2%
b. 11.3% d. 18.1%

30. On a promotional exam a woman scored 143 on a scale of 0-160. Her score converted to a scale of 0-100 is approximately:

a. 89 c. 91
b. 70 d. 84

31. A woman paid a tax of $88.00 on property assessed at $28,000. Her neighbor, assessed at the same rate. paid a tax of $110. What was the assessed value of the neighbor's house?

a. $22,400 c. $35,000
b. $32,400 d. $31,000

32. If Janet can build 22 tables in 14 days, and Anne can build 22 tables in 16 days, approximately how long will it take them to build 22 tables together?

a. 9.5 days c. 15 days
b. 7.5 days d. 8 days

33. Cynthia loaned $35 to Mary. But Cynthia borrowed $14 from Jean, and $16 from Emily Emily owes $17 to Jean and $9 to Mary. One day they got together to settle their accounts. Who left with $10 less than she came with?

a. Cynthia c. Mary
b. Jean d. Emily

34. How many square tiles, each 12 inches on a side, will Ozzie need to cover a floor that is 11 feet wide and 18 feet long?

a. 99 c. 163
b. 150 d. 198

35. A car has depreciated to 72% of its original cost. If the car is presently valued at $3245, approximately what was its original cost?

a. $5219 c. $4507
b. $5582 d. $2336

36. The sales tax on a typewriter is $13.41 and the sales tax rate is 4%. The purchase price, before the tax was added, was:

a. $335.25 c. $279.10
b. $536.40 d. $317.50

37. What is the interest on $600 at 8% for 30 days?

a. $4.00 c. $7.50
b. $11.52 d. $4.80

38. A garden is 30 feet by 40 feet. A fence is built around the garden at a cost of $1.75 per foot of fencing. What was the cost of the fencing?

a. \$133.33 c. \$210.00
b. \$245.00 d. \$122.50

39. A tax analyst earns four times as much in April as in each of the other months. What part of her entire year's earnings does she earn in April?

a. 4/11 c. 4/15
b. 1/3 d. 4/13

40. A train travels 70 miles when a bus travels 50 miles. How many miles will the train travel when the bus travels 60 miles?

a. 40 c. 90
b. 78 d. 84

METRIC ARITHMETIC REASONING

1. How many square tiles, each 300 mm on a side, will Jane need to cover a hall floor that is 120 cm wide and 3.6 m long?

a. 480 b. 520
c. 490 d. 48

2. A property is 100 m by 0.337 km. A fence is built around this property at a cost of \$1.75 per foot of fencing. What wm the cast of the fencing?

a. \$5,018 b. \$764.75
c. \$502 d. \$2,009

3. A car travels 80 km/hr and a plane travels 16 000 m/min. How far will the car travel when the plane travels 800 km?

a. 80.6 km b. 66.7 km
c. 60.0 km d. 63.5 km

4. A pharmacist combines ingredients x, y, and z in a ratio of 2:3:5 to produce cough medicine How many milliliters of the second ingredient, ingredient y, is needed to make

a one liter bottle of the medicine?

a. 0.3 b. 150
c. 300 d. 500

5. A cabinet maker has a dowel of wood 13 mm in diameter and 0.800 m long. She needs half the length for the back of the chair and the remaining piece for 37 mm pegs. Pegs greater than 13 mm in diameter or shorter than 37 mm will not work. How many good pegs can she make?

a. 108 b. 11
c. 10 d. 109

ANSWER KEY

ARITHMETIC REASONING

1.	c	21.	b
2.	c	22.	a
3.	d	23.	b
4.	a	24.	d
5.	b	25.	c
6.	b	26.	d
7.	d	27.	c
8.	c	28.	b
9.	a	29.	a
10.	d	30.	a
11.	b	31.	c
12.	a	32.	b
13.	c	33.	d
14.	a	34.	d
15.	d	35.	c
16.	b	36.	a
17.	c	37.	a
18.	a	38.	b
19.	a	39.	c
20.	c	40.	d

METRIC ARITHMETIC REASONING

1.	d
2.	a
3.	b
4.	c
5.	c

SELF STUDY GUIDE

ARITHMETIC REASONING

You should consult this guide whenever you miss a question or aren't sure why you got the answer you did.

You shouldn't get discouraged if you seem totally lost at first. With practice you will improve. In many cases these questions require using methods you may not have used in years, if ever. We have tested this, guide with many people, however, and all of them have been able to improve their ability to answer word problems by conscientiously using it. We don't mean to suggest that sometimes it won't be hard work - you may need to re-work and re-read some of the problems many times before they make sense. You will get out of this guide the fruits of whatever effort you put in, and perseverance in problem solving is always critical.

No knowledge of advanced math is required, and we have kept our explanations free of jargon and intimidating formulas. Basically, what you need is a knowledge of basic math and perseverance. In explaining the answers, we briefly review working with fractions, percentages and ratios.

It's also important to remember that there are often many ways to do a particular problem. We are presenting methods that are the easiest for most people. If you have a different approach, and you consistently get the right answer using it, there's certainly no need to change.

Good luck!

ARITHMETIC REASONING

1. The answer is C. This kind of question is difficult unless you break it down into parts, and solve it step by step. It's only in the next to last sentence that we're actually given someone's salary. We're told that Betty makes $200 a week. The question asks for Barbara's salary, but it becomes clear after careful reading that we can't find Barbara's salary unless we find Jean's. Jean's weekly income is $80 more than half of Betty's.

   Jean's salary is $80 more than half of $200, so Jean's salary is $80 more than $100, or $180. The first sentence tells us that if Jean's weekly income doubled she would be making $120 a week more than Barbara. Two times Jean's salary of $180 would be $120 a week more than Barbara's salary. So, two times $180, which equals $360, would be $120 a week more than Barbara's salary. $360 is $120 more than what number? $360 - $120 - $240, Choice C.

2. Choice (C) is the answer: It's important in a question like this to identify and break down

the information you are given. From the question, we know:

1. 3600 people are attending the conference

2. 1/2 of the 3600 people have ordered meals

3. One out of 12 (or 1/12) of those who ordered meals has special dietary needs

It's important to remember that in order to solve this we must keep in mind that only half of the attendees have ordered meals. So half of the 3600 people, or 1800, have ordered meals. Of these, one out of every twelve has special dietary needs, so 1/12 of the 1800 people who signed up for meals have special dietary needs. There are many ways to solve this problem from this point on. "One way is to simply multiply 1800 by 1/12 to find the answer. (To multiply fractions, multiply the numerators by each other, and the denominators by each other.)

$$1 / 12 \times 1800 = 1 / 12 \times 1800 / 1 = 1800 / 12 = 150$$

So, 150 of those who have ordered meals have special dietary needs, Choice (C). Or you could have used decimals. 1/12 is expressed in decimal form as .0833 (to find the decimal form of a fraction, divide the numerator, the top number, by the denominator, the bottom number). 1800 x 1 / 12 = 1800 x .0833 = 149.94 or 150 people. (It comes out a little unevenly because the decimal has been rounded off.) You also could have set up a ratio, comparing those with special needs who ordered meals to all who ordered meals.

A Review of Ratios

Ratios are intimidating for many people in an exam setting. Yet, we use ratios in "real life" - inches to miles on a map, or the ratio of ingredients in recipes in cooking. A ratio shows the relationship between two numbers. In this case, it shows the relationship between those who have dietary restrictions and ordered meals to all those who ordered meals. One out of every twelve people who have ordered meals has special dietary needs, so we need to examine the relationship between the numbers one and twelve, and apply it to the 1800 people who have ordered meals.

$$\frac{\text{Special dietary needs}}{\text{All who ordered meals}} \quad \text{as} \quad \frac{1}{12} \quad \text{as} \quad \frac{\text{what number}}{1800}$$

One way to do this is to cross multiply. To cross multiply, we multiply the top of one number by the bottom of the other.

$$\frac{1}{12} = \frac{?}{1800} \qquad \frac{1}{12} \quad \frac{?}{1800}$$

12 x what number ? = 1 x 1800
12 x ? =1800
? = 1800 / 150 = 150

(You have to divide the 1800 by the 12, because you want to "isolate" the ? on one side, since that will give you the answer. Since the 12 and the ? were being multiplied by each other, the only way you could "free" the ? was to move the12 over to the other side of the equal sign by dividing the 1800 by the 12. If you get as far as 12 x ? = 1800 but still can't remember whether you should multiply or divide, you can still get the correct answer, because the difference between multiplying add dividing is so large that common sense will tell you which is right. In this case, dividing gives you 150 people, and multiplying gives you 270,000 (obviously too big a number). So, in a ratio problem, as long as you set up the relationship between the numbers involved correctly (part is to whole as part is to whole), you should be able to solve it.)

Another way some people do ratios is by remembering that "the product of the means equals the product of the extremes". This means that when you multiply the 'Inside' numbers in a ratio problem together, and then multiply the "outside" numbers together, they will always equal each other. In this case, we would set it up like this-

1 is to 12 as ? is to 1800

The "inside" numbers, 12 and ?, would be multiplied together, 12 x ?, and would equal then "outside" numbers that have been multiplied together, 1800 and 1. So

12 x ? = 1800 x 1
12 x ? = 1800
? - 1800 / 12 = 150

All of this may have seemed totally unnecessary, but it's important to keep these methods in mind for other questions, when using them may be very helpful. Again, there are many ways to solve math problems. If you use different methods than those in this booklet, and your results are consistently correct, there's no need to change what you're doing.

3. The answer is D. This problem is actually easier than it may look at first, and there are a number of ways to do it. One way would be to first determine how much the office is paying per typewriter. We're told that it costs \$360 to service 18 typewriters for six months. So, for a six month period, it would cost, per typewriter, the total amount, \$360, divided by the total number of typewriters, 18. \$360 ÷ 18 = \$ 20 per typewriter. So it costs \$20 to service each typewriter for a six month period. We need to find out how much it would cost to service six typewriters for a three month period. We know the service cost of each typewriter is \$20 for six months. For three months, it would be half that amount, or \$10 per typewriter. Since we are considering six typewriters, the cost

would be six typewriters at \$10 each, for a total of 6 x \$10 = \$60.

Or we could have said that we're being asked to find the cost of one third of the typewriters (6 is 1/3 of 18), for half the time (3 months is half of six months). One third of the typewriters would cost 1/3 as much: 1/3 x \$360 = \$120, and since the time involved is 1/2 of the total time, 1/2 of \$120 equals \$60. or we could have set it up using fractions: $360 \times 1/2 \times 1/3 = 360/6 = 60$.

4. The answer is A. Many people miss this question, because they aren't quite sure what to do with the 15% figure they're given. We are told that \$480 is 15% of the office's non-personnel expenses for December. So in order to find the answer we need to know what number 480 is 15% of. So, we are asking "480 is 15% of what number?". To find this, we divide 480 by 15%.

$$480 \div 15\% = 480 \div .15 = 3200$$

It's always a good idea to go back to the problem and check our answer to see if it makes sense. Is 480 15% of 3200? If we divide 480 by 3200, we get .15 = 15%. So 480 is 15% of 3200, and if we multiply 3200 by .15, we get 480.

Some people aren't sure whether to multiply or divide. Again, common sense should tell you which, in this case, if you had multiplied, you would have gotten \$72 as an answer, which doesn't make much sense. It's important in these exams to step back and evaluate the reasonableness of your solutions , yet people often fail to do this.

It was also possible in this question to work backwards from the answers given to get the right answer. If you weren't sure how to do it, you could have multiplied each choice by 15%, to see which one was equal to \$480. Choice A, 3200 x .15 = 480, would have become the obvious choice. This is a legitimate way to solve these types of problems Some people select Choice D, because they incorrectly multiply, and then aren't sure where to put the decimal points. If you get confused, we suggest you use the sales tax to help you remember. For example, a 7% sales tax reflects a tax of \$.07 on every dollar. in the corner of your scrap paper you could write, 7% = .07; .07 = 7% or 8.25% = .0825; .0825 = 8.25%. (If you do this, when a percent like .0035% comes along on a word problem, you'll be able to convert it to decimal from more easily, especially if you're nervous. Consulting the sales tax example, we'd notice that 7% = .07 meant the decimal was moved two places to the left when going from percents to decimals, so .0035% would equal .000035. Or, if we had to convert a decimal like .00046 to a percent, consulting the sale tax, we'd see that in the case of .07 = 7%, the decimal was moved two places to the right, so we'd do the same here. .00046 would then equal .046%. Sorry if this was unnecessary, but many people get confused when dealing with decimals and percents on an exam).

5. The answer is B. Many people put D as an answer, thinking that it's not possible to find the answer from the information given. The key thing to note here is that the question says Catherine bought an equal number of tickets. That means that the relationship between each of the ticket amounts will always be equal. There won't be more $11 tickets than $9.00 or $8.00 tickets. (It's true that if the question didn't state there were equal amounts of tickets, D would be the correct answer). Since we know there are equal amounts of each ticket, however, we can find the answer by: 1) adding up the cost of the tickets and 2) dividing this figure into the total dollar amount she paid for them. $11 + $9 + $8 = $28. She spent a total of $196, so we can find out how many of each she bought by dividing 196 by 28. 196 ÷ 28 = 7. Some people aren't sure how to do this problem at first but by spending time looking at the problem and playing with all the possible choices, it becomes clearer.

6. The answer is B. The first thing we need to do is find the number of males in Agency Y. Agency Y employs 13,800 people, 42% of whom are male. So we would multiply 13,800 by 42% to find the number of males. 13,800 x 42% is 13,800 x .42 = 5796 males in Agency Y. Of these, 50% are age 30 or younger. So half of the 5,796 males will be older than 30. 5,796 ÷ 2 = 2898 males older than 30.

7. The answer is D. There are manyways you could solve this problem. This is another ratio problem (see question #2). We're told a machine can collage 126 books of 400 pages each in 14 days. We need to find how many books it could collate in 30 days. Since it collates at the same rate, the answer will be in the same proportion as 126 is to 14.
So, ? / 30 = 126 / 14 There are many ways to do this.

One is to first find the relationship between 14 and 126. 126 is nine times 14, so the answer will be 9 times 30, or 270. Or, you could set up a ratio and cross-multiply (see #2).

$$\frac{?}{30} = \frac{126}{14}$$

$$\begin{aligned} 14 \times ? &= 126 \times 30 \\ 14 \times ? &= 3780 \\ ? &= 3780 \div 14 \\ ? &= 270 \end{aligned}$$

Or. you could have used "the product of the means equals the product of the extremes." (See Question #2)

14: 126 as 30: ?

$14 \times ? = 126 \times 30$

$14 \times ? = 3780$

$? = \frac{3780}{14}$

$? = 270$

8. The answer is C. Many people miss this question, because it's more difficult than it looks. Many will simply subtract \$360 from \$840, and get \$480, choice A. Yet, if we go back through the problem and re-read it, we'll see that choice A can't be the correct answer. We know the typewriter and dictation machine together cost \$840. We're also told that the typewriter cost \$360 more than the dictation machine. So, to find the cost of the typewriter, and to check choice A, we should be able to add \$360 to choice A's \$480 to find the cost of the typewriter. \$360 + \$480 = \$840. So, according to choice A, the typewriter costs \$840. The total cost of the typewriter and dictation machine then becomes \$360 + \$840 = \$1200. But the question states the totla of both was \$840, so we know something is wrong. We're looking for an answer that will total \$840 for the two objects, and that will also show that the typewriter is \$360 more. In choice A, the typewriter comes out to be \$360 more, but the total is not \$840. There are several ways you can do this problem. A very legitimate way is to "work backwards" from the possible answers. You can take each choice, add \$360 to it, and see if they total \$840. If you do this, it becomes apparent that choice C is the answer. Adding the typewriter, \$600, and the dictation machine, \$240, they total \$840. The \$600 typewriter is \$360 more than the \$240 dictation machine, so it checks out. This is a perfectly acceptable way to solve the problem.

There are many other ways to solve it. One is to use a little algebra.

$x + (x + 360) = 840$

$2x + 360 = 840$

$2x = 840 - 360$

$2x = 480$

$x = 240$

You don't need to know algebra to solve it, though. Another way to solve a problem like this is to take the difference between the total of the two numbers and the difference between them, and then divide by two. This will always give you the smaller number.

840 - 360 = 480. 480 ÷ 2 = 240. This method will always work, as it's the same mathematical operation used in algebra, without the algebra. Working backwards from the answers given will also always work.

9. The answer is A. This question looks more difficult than it is, primarily because unnecessary information is thrown in. The diameter of the wood is not needed. We know that half the length of the wood is needed for the back of the chair. Because the rest of the problem is given in inches, the easiest approach would be to convert 3/4 of a yard into inches. 3/4 yard equals how many inches? 3/4 yards equal 27/36 or 27 inches. We need to find what half of 27 inches is to find how many inches will be used for pegs. $27 \div 2 = 13\ 1/2$ inches. Since 13 1/2 inches are being used for the back of the chair, 13 1/2 inches are left for the pegs. Since each peg is 3/4 of an inch, we should divide the total length available, 13 1/2 inches, by 3/4 of an inch to find how many pegs can be made. $13\ 1/2 \div 3/4 = 27/2 \div 3/4$ (to divide fractions, we invert the second fraction and then multiply) $27 / 2 \times 4 / 3 = 108 / 6 = 18$ pegs. Or, if you hate fractions, you could have converted to decimals and then divided.

10. The answer is D. There are several ways to do a problem like this. One way to do this is to remember the following method. First, invert the two numbers you're given, four hours and seven hours. So these numbers become 1 / 4 and 1 / 7. Then, add them together. (1 / 4 + 1 / 7 do not equal 1 / 11; we have to find a common denominator first). Twenty-eight is a number both four and seven will divide evenly into.

$$\frac{1}{4} = \frac{7}{28} \quad \text{and} \quad \frac{1}{7} = \frac{4}{28}$$

$$\frac{7}{28} + \frac{4}{28} = \frac{11}{28}$$

The last thing needed is to invert again to find the answer. $28 / 11 = 2.545$.

If you remember this method, invert, add together, and invert again, you will always be ably to answer this type of question. The problem can also be solved algebraically. Let x be the time it takes. Robin can wallpaper a room in 4 hours, and Susan can wallpaper a room in 7 hours. In x hours, Robin does x / 4 part Of the work, and Susan does x / 7 part of the work. Together they do the complete job.

$$\frac{x}{4} + \frac{x}{7} = 1.$$

(Multiply the equation by 28 to get rid of the fractions)

$$7x + 4x = 28$$
$$11x = 28$$
$$x = 28 / 11 = 2.545$$

Sometimes it's also possible to estimate an answer in these types of questions. When that isn't possible, it's good to use the first method or the algebraic method, as they always work with this type of work problem.

11. The answer is B. Most people find these types of questions irritating, not only because they bring back bad memories, but also because of their basic irrationality - who ever jogs, (or wallpapers, or builds chairs) at exactly the same rate every time. Nevertheless, it's important to know how to do these. In this problem, we need to first find Mary's speed. We can do this by dividing the distance she traveled (3 miles) by her time (40 minutes). $3 \div 40 = .075$ miles per minute. We know Mary runs for 40 minutes. Alice runs the same distance in 30 minutes, so Mary has 10 minutes of running time left when Alice finishes. To find how much distance Mary has left, multiply her speed, or rate by the time left.

$.075 \times 10 = .75$, or 3/4 of a mile.

12. The answer is A. We first need to determine how much the organization spent on each group of tickets.

50	\$8 tickets	= 50 x 8	= \$400
25	\$10 tickets	=25 x l0	=\$250
25	\$15 tickets	=25 x15	=\$375

Since all the \$8 and \$15 tickets were sold at 25% above cost, the money spent on these tickets came back and the organization made a profit of 25%. So the total spent on these two groups of tickets was \$400 + \$375 = \$775.
We need to find the 25% profit on these.

25% of 775 = .25 x 775 = \$193.75.

\$193.75 is the profit from the \$8 and \$15 group of tickets. Of the 25 ten dollar tickets, three were unsold. Since each ticket was worth \$10, and there were three unsold, \$30 was spent by the organization in buying these tickets that did not come back. So the \$30 will have to be subtracted from what ever profit is made. We should then find the profit on the 22 ten dollar tickets that did sell. 22 x 10 = \$220. 25% profit on \$220 = .25 x 220 \$55.00. There was a \$55 profit on the ten dollar tickets sold. But we need to subtract the \$30 worth of unsold tickets from this. 55 - 30 = \$25 profit on ten dollar tickets, Remember that the organization made \$193.75 on the other ticket sales. Adding \$25 to \$193.75, the total profit was \$218.75.

13. The answer is C. Another percent problem. There are many ways to do this. One way to do this is to ask, \$640 is 80% of what number? 640 / .80 = 800. (If you're not sure whether to multiply or divide, you could still figure it out. If you multiply you should notice that the answer you get, \$512, is less than \$640, so it couldn't be correct). Another way to do it would have been to set up a ratio.

$$\frac{640}{?} = \frac{80}{100}$$ (80% is to 100%)

$80 \times ? = 640 \times 100$, $80 \times ? = 64{,}000$, $? = 64{,}000 / 80$

? = 800. Or, you could work backwards and go through each choice, multiplying by 20% and then subtracting this amount, to see which choice would give you $640. Choice C is $800. $800 x .20 = 160. $800 - $160 = $640.

14. The answer is A. It's first necessary to determine the total amount spent on travel costs in the three days. The total mileage was 145 + 72 + 98 = 315. The salesperson traveled 315 miles, at 21¢ per mile. So the cost is 315 x 21 = $66.15. We need to find what percent the travel costs were of sales. To do this we divide $66.15 by $2300.

    66.15 ÷ 2300 = .0287 = 2.87%.
    Choice A, 3% is the closest of the four possible choices.

15. The answer is D. This is a tricky question because many people assume they must set up a ratio, and find the number of students. But the question is asking for the percent of students, not number of students. One out of every 8 students takes Latin. The number 650 is irrelevant. All we need to find is what percent 1 is of 8. 1 ÷ 8 = .125 = 12.5%. 12.5% is closest to choice D, 13. (They don't need to put a percent sign next to each choice, as the question is asking for an answer in percents.) Once again, this question shows the importance of reading the problem carefully.

16. The answer is B. We first need to determine the total value of the in-kind contributions by adding $14,500 and $1,200. 14,500 + 1,200 = $15,700 . Then we need to find what percent of the total of $102,000 the in-kind total of $15,700 is. We can do this by dividing the in-kind contributions by the total budget.

    15,700 / 102,000 = .1539 = 15.4%

17. The answer is C. We know from question #16 that the program has a budget of $102,000. We can find the answer by setting up a relationship between the pounds of food distributed to the money budgeted.

    $$\frac{\text{lbs. of food}}{\text{money budgeted}} = \frac{250{,}000}{102{,}000} = 2.45$$

    There were 2.45 pounds of food distributed for each dollar spent.

18. The answer is A. One way to do this is to first find how many cases of jars were purchased. Jars are sold in cases of 12, not individually. If there are 76 quarts of tomatoes, dividing 76 by 12, we get 6.33 cases. Since we can t buy jars individually, we need 7 cases. For 20 pints of jelly, we'll do the same, dividing 20 by 12. 20 ÷ 12 = 1.66, so we'll need 2 cases. We'll need a total of 9 cases of jars. The regular price per case of

jars is \$4.25. But there is a discount of 15%.
15% of 4.25 = .15 x 4.25 = .637 = .64 (to the nearest cent).
\$4.25 - the discount of .64 = \$3.61 per case. There are 9 cases that are purchased at \$3.61 per case. So the cost of jars alone is \$3.61 x 9 = \$32.49. Next we have to find the cost of the processing. The cannery charges 15¢ per quart. There are 76 quarts at 15¢ per quart, so the cost of processing equals 76 x .15 = \$11.40. There are 20 pints that need to be processed at a cost of 25¢ for 2 pints, so the cost of processing will be 20 x .25/ 2 = \$2.50. So, the cost of the canning will be the total of the cost of the jars, \$32.49, the cost of processing 76 quarts , \$11.40, and the cost of processing 20 pints , \$2.50. \$32.49 + 11.40 + 2.50 = \$46.39.

19. The answer is A. This is a different kind of ratio problem. There are several ways to solve it. One way is to first add the parts given in the ratio in the problem. 1+ 2 + 7 =10. Then divide this number into the total amount of whatever substance you've been given. In this case, its 12 ounces of cough medicine. This will give the value of each part. 12 ÷ 10 = 1.2. Now, to find how many ounces of each ingredient is used, we would multiply 1.2, which represents one part, by the ratio of each of the ingredients given. Ingredient X is worth 1 part, so ingredient x = 1.2 x 1 = 1.2 oz. Ingredient Y is worth 2 parts, so 1.2 x 2 = 2.4 oz. Ingredient Z is worth 7 parts, so 1.2 x 7 = 8.4 oz . We can check to see if adding them would give us 12 ounces. 1.2 + 2.4 + 8.4 = 12.0 ounces, so it checks out. The question asks us for the amount of ingredient Y, 2.4 oz. (choice A). Another method is to express this algebraically: A + 2A + 7A = 12

10A = 12 A = 12 / 10 A = 1.2 2A= 2.4

Or, you could work backwards from each choice, but in this case it's more work than using the above methods.

20. The answer is C. This is a percent increase question. We're told that the people served by Agency Y increased from 187,565 to 210,515. We need to find the percent increase. TO FIND PERCENT INCREASE OR DECREASE: 1.)Take the difference between the two numbers being considered, and 2.) Divide this difference by the original number, the number that chronologically came first. The difference between 210,515 and 187, 565 is 22, 950. 22,950 divided by 187, 565 (the earlier 1981, figure) equals .122 = 12.2%. If you can remember these two steps, you will always be able to answer this type of question.

21. The answer is B. There are several ways to do this. If, while doing these problems you use different methods, you shouldn't worry as long as your getting the right answers. There are many ways to approach these problems. One way to do this would be to set up an equation.

$$\frac{105 + 106 + 125 + ?}{4} = 116$$

The average of the three known numbers, and the unknown number equals 116. The above equation shows that if we add the four numbers together, and then divide by 4, we'll get 116. The 4 as a divisor is cumbersome. To get rid of it, we can multiply each side by 4.

$$\frac{4 \times (105 + 106 + 125 + ?)}{4} = 116 \times 4$$

$$105 + 106 + 125 + ? = 464$$

$$336 + ? = 464$$

$$? = 464 - 336$$

$$? = 128$$

You can check this if you wish by adding: 105 + 106 + 125 + 128 = 464. Dividing by 4, we get 116, so it checks out. Or, you could have solved this problem by working it backwards, taking each of the possible choices, adding it to the other three numbers, and then dividing by four to see if their average was 116. If you did this, which is a perfectly legitimate way to solve problems of this type, you would also have gotten 128, Choice B.

22. The answer is A. This problem can be solved without a lot of difficulty if the relationships between the workers and their times is kept clearly in mind. If it takes 16 typists 11 days to complete a project, we need to find how long it will take 10 typists working at the same rate. The 10 typists would complete the job 10/16 as quickly. So we could find the answer by dividing the days it took 16 typists, 11 days by 10/16.

$$11 \div 10/16 \quad = \quad 11 \div 5/8 \quad 11 \times 8/5 \quad = 88/5 \quad = 17.6 \text{ days.}$$

If you weren't at all sure how to do this, you may have wanted to first use a simpler example, so that you could then visualize what needed to be done. For example, what if the question had read "It takes 4 typists 8 days to complete a project. It would take 2 typists how many days?" You would have figured out that 2 typists would take twice as long, so it would have taken them 16 days. If you examined how you got this answer more carefully, you would be able to derive a method that could be used to solve the question. 2 typists is half of four typists. The 2 typists would complete the job 2/4 or half as quickly. The number of days it took was 8. You would then divide 8 by 1/2 to get the answer. $8 \div 1/2 = 8 \times 2/1 = 16$. If you weren't sure what you were supposed to do at this point, multiply, divide, or whatever, yet had a clue in that you knew the answer was 16. So you would do whatever would give you 16, and that was divide. This is a legitimate way to solve a problem, using a simpler, clearer relationship between two numbers, seeing how the problem would be solved, in that case and then applying the method to the test question. If you're stuck on how to approach a question, it's a good way to gain insight into how to solve it.

23. The answer is B. This is the same type of ratio problem as Question 19. The first thing we need to do is add the parts of the ratio together. 7 + 3 . 10. We then divide this into the total of the two numbers, 280. $280 \div 10 = 28$. This means each part is equal to 28. The smaller number will equal 3 parts of 28, and the larger will equal 7 parts of 28.

$$3 \times 28 = 84. \qquad 7 \times 28 = 196$$

We're asked for the smaller number, 84. We can check this by adding 84 and 196 to see if they equal 280. 84 + 196 = 280.

24. The answer is D. Another percent problem. The 1982 population of Metropolis county is 130% of its 1972 population, 145,000. So the population will be 130% x 145,000. (130% = 1.30). 1.30 x 145,000 = 188,500, Choice D. (If you weren't sure whether to multiply or divide, division would have given you a smaller number than the 1972 figure, Choice C, which wouldn't make sense since there was an increase, not a decrease, in population).

25. The answer is C. We know that the plane travels 10 miles a minute and the car travels 50 miles an hour. To find how far the car will travel when the plane travels 500 miles, we need to first find out how long it will take the plane to travel 500 miles. At 10 miles a minute, the plane will take 500 miles divided by 10 miles a minute,

$$\frac{500\ \cancel{\text{miles}}}{10\ \cancel{\text{miles}}/\text{ minute}} = 50 \text{ minutes.}$$

We need to find how far the car has traveled in 50 minutes. If the car travels 50 miles in 60 minutes, how far will it travel in 50 minutes? We can set up a ratio to find this.

$$\frac{50 \text{ miles}}{60 \text{ minutes}} \quad \text{as} \quad \frac{? \text{ miles}}{50 \text{ minutes}}$$

One way to solve it is to notice that since this is a ratio problem, these numbers will be in direct proportion to each other. 50/60 is 5/6, So the answer will be 5/6 of 50.
5/6 x 50 = 41.66

26. The answer is D. We first need to find the total number of faculty. We know the ratio of students to faculty is 16:1, and there are 2,000 students. So we can find this by setting up a ratio.

$$\frac{\text{Students}}{\text{Faculty}} \quad \text{as} \quad \frac{\text{Students}}{\text{Faculty}} \qquad \frac{16}{1} \qquad = \qquad 2000$$

You can solve from here in a number of ways. One way is to observe that 2000 is 125 times greater than 16, so what we're trying to find will be 125 times greater than 1. Or, cross multiply:

16 x ? = 2000 x 1
16 x ? = 2000
? = 2000/16
? = 125

Or, use "the product of the means equals the product of the extremes", (see Question #2).

16 : 1 = 2000 : ?
16 x ? = 2000 x 1

$$? = 2000/16$$
$$? = 125$$

We know there are 125 faculty. If approximately 18% of them studied at the university then 82% did not. We need to find 82% of 125. 125 x .82 - 102.5 (Or you could have multiplied 125 by 18% and then subtracted the result from 125).

27. The answer is C. One of the few good things about multiple choice math questions is that the answer has to be one of the four given. In a problem like this, if you can't figure it out, it's possible to work backwards to get the answer. Most people select Choice A, because they misread and think $30 is the ten percent profit of the sales price of $300. But she bought the set for $300, and the question states that they sell all merchandise at cost plus 10%. If the profit was $30, the set would have to cost $300. But this would mean the total cost would be $330 . The cost, plus $30 (the 10% profit) added on, would total $330, not $300. So Choice A can't be correct. One way to do this would be to work backwards from each choice given. Choice B states that $27 is the profit. $27 is 10% of $270. Added together they equal $297, not $300, so Choice B is incorrect. Choice C is $27.27. $27.27 is 10% Of $272.70. $272.70 + 27.27 = $299.97. Since the answer says approximately, this looks like a safe choice. But just in case if you're not sure, we can check Choice D. $32.26 is 10% of $322.60, so we know that they won't add up to $300. Choice C is the answer. This is a perfectly good way to solve this problem. Arithmetic Reasoning is also testing your resourcefulness at working with numbers, and working backwards if you're stuck is certainly being resourceful. You could also say to yourself, $300 is 110% (100% is the cost of the item, plus a 10% profit added on) of what number? 300 ÷ 110% will give us the answer. 300 ÷ 1.10 = $272.72. So we know $272.72 is the actual cost. The profit will be 10% of this, or $27.27. Or you could have set up a ratio: 300/? = 110/100 (110% is to 100%) These methods are quicker than working backwards, so you may want to spend some time studying them.

28. The answer is B. We know that the temperature dropped 8° from midnight until 1 a.m., and before this it dropped at a constant rate. Working backyards from 1 a.m., we can add 8° to the temperature given at 1 a.m., 37°, to find the temperature at midnight. 37 + 8 = 45° at midnight. The temperature dropped at a constant rate from six p.m. to midnight. During that time it went from 54° to 45°. This is a drop of 9°. 54° - 45°, in 6 hours elapsed time from 6 p.m. to midnight. In 6 hours, the temperature dropped 9°. To find the rate the temperature dropped each hour, we would divide 9° by 6. 9° ÷ 6 = 1.5°. Since the temperature dropped 1.5° each hour, we can find the temperature for 10 p.m. by subtracting (4hours times 1.5 degrees), which equals 6° from 54. 54 - 6 = 48°, the temperature at 10 p.m. Or we could add two hours times 1.5 degrees, 3° on to the midnight temperature of 45°, 45 + 3 = 48°, Choice B.

29. The answer is A. We know that one eighth of a half gallon carton of ice cream has been

eaten, and the remainder is divided by three people. The trick to this question, and it's a tricky question, is that they are asking for what percent of a gallon each, person gets, not of a half gallon, and there is only a half gallon of ice cream to begin with. So if 1/8 of a half gallon has already been eaten, we can find out how much this is by multiplying 1/8 by 1/2. 1/8 x 1/2 = 1/16. So 1/16 of a gallon has been eaten. The remainder is divided by three people. There was a half gallon, but 1/16 has been eaten. That leaves 1/2 - 1/16 • 8/16 - 1/16 = 7/16 left. The remainder, 7/16, is divided by 3 people.

7/16 ÷ 3 = 7/16 x 1/3 = 7/48

So each person gets 7/48 of a gallon. But the answer has to be expressed in percents. 7/48 as a percent is 7 divided by 48 =.1458, = 14.6%, Choice A. Fortunately, few of the exam questions are this tricky. It's always good to re-check your answers, with this type of question.

30. The answer is A. This question looks more difficult than it is. We know the woman got a score of "143 on a scale of 160". This means that out of 160 questions, she got 143 correct. We're asked to convert her score "to a scale of 0-100". All that means is that we're going to convert her score into a percent. (Percents are based on 100.) 143 is what percent of 160? To find this, we divide 143 by 160. 143 ÷ 160 = .893, or 89.4, Choice A.

31. The answer is C. This is another ratio problem. Since the assessment is at the same rate, we can set up a ratio between the numbers involved. (See Questions 2, 25, 26.)

$\frac{\text{tax}}{\text{value}}$ as $\frac{\text{tax}}{\text{value}}$ $\qquad \frac{88}{28{,}000}$ as $\frac{110}{?}$

or $\frac{\text{tax}}{\text{tax}}$ as $\frac{\text{value}}{\text{value}}$ $\qquad \frac{88}{100}$ as $\frac{28{,}000}{?}$

You can then solve it. If you cross multiply:

$\frac{88}{110} = \frac{28{,}000}{?}$

88 x ? = 28,000 x 110
88 x ? = 3,080,000
? = 3,080,000/88
? = 35,000

32. The answer is B. Here's another work problem (see question 10). For the type of work problem that asks you to combine the efforts of two different people, there are several approaches you can use. One way to do this is to first invert the two numbers you're given, 14 days and 16 days. They become 1/14 and 1/16. Then, add them together. You'll need to find a common denominator to do this. 1/14 + 1/16 = 8/112 + 7/112 = 15/112. Then invert again to find the answer. 112/15 = 7.466 days.

33. The answer is D. This is another tricky question. It's important to remember that they got together to settle their accounts. This means they were paying each other back. It's a good idea to break this problem down into parts. A good way to do it is to write each

person's name out, with the amount they are paying back or receiving directly under the name. So, Cynthia loaned \$35 to Mary. Since they've gotten together to pay each other back, Cynthia will be getting back \$35 from Mary. So we'll put +35 under Cynthia's name, and a -\$35 under Mary's. Cynthia borrowed \$14 from Jean: -14 for Cynthia (she's paying Jean back), +14 for Jean. She also borrowed \$16 from Emily, so -16 for Cynthia, + 16 for Emily. Emily owes \$17 to Jean: -17 for Emily, + 17 for Jean. Emily owes \$9 to Mary: -9 for Emily, +9 for Mary.

	Cynthia	Jean	Mary	Emily
	+35	+14	-35	+16
	-14	+17	+9	-17
	-16			-9
Total	+5	+31	-26	-10

Emily left with \$10 less than she came with. A lot of work for a point and a half.

34. The answer is D. We know the tiles are square, and that they are 12 inches on each side. To find out how many we'll , we need to find out how large the floor is. We're told it's 18 feet long and 11 feet wide. Multiplying 18 by 11 will give us the area that needs to be covered. 18 x 11 = 198 square feet. Since each tile is exactly a foot on each side, we'll need 198 of them.

35. The answer is C. Another percent problem. We know the car is presently valued at \$3245, and that it's worth 72% of its original cost. To find this, we could say "72% of what number is \$3245?"

$$72\% \times ? = 3245$$
$$.72 \times ? = 3245$$
$$? = 3245/.72$$
$$? = 4506.94$$

Check it: is 3245 72% of 4506.94? 3245 ÷ 4506.94 = .72, or 72%.
Or you could have set up a ratio.

3245/? = 72/100 (72% as to 100%) Or you could have worked backwards, taking 72% of each of the possible answers to see which would give you \$3245.

36. The answer is A. Again, a percent problem. They're very common on this section. We know the sales tax rate is 4%, and the tax on the typewriter was \$13.41. To find the purchase price before the tax was added, we could ask "13.41 is 4% of what number?" 13.41 is .04 of what number?

$$4\% x ? = 13.41$$
$$.04 \times ? = 13.41$$
$$? = 13.41/.04$$

BASIC ALGEBRA

Algebra evolved from a simple and useful idea. When referring to an unknown number, it was more convenient to use a label or symbol to represent that number than to keep writing or saying "an unknown number." Mathematicians have given the unknown number a name, so that they can distinguish it from other numbers more easily. The most commonly used symbol or name for an unknown number, for our purposes, is x. But many letters of the alphabet (in Greek, Latin, italics, and lower and upper case) are used in algebra and statistics to represent or name both known and unknown numbers. Although it initially makes learning this material more difficult, once you've learned how to work with these combinations of letters and numbers, mathematical operations needed for word problems and statistics really do become easier to do.

In the first part of this booklet, we're going to be outlining all the rules and procedures you'll need to do basic algebra. It may all seem strange and irrelevant, but we'll be giving you the structural "building blocks"of algebra. Without these, it will be impossible to do the algebraic and statistical problems that appear on exams. We've also included a review of working with positive and negative numbers, since this knowledge is critical. We won't be getting to working with algebraic expressions, equations or statistics until much later in the booklet. Hopefully by that time you'll have a thorough understanding of the algebraic "basics", and you'll, be able to work with these much more easily. If some of this material seems familiar, please bear with us; we've designed this assuming no knowledge of algebra on the part of the reader.

SYMBOLS FOR THE FOUR FUNDAMENTAL OPERATIONS USED IN ALGEBRA

Except for multiplication, the symbols used in algebra are the same for thse used in regular math. It's very important to remember these.

Addition:	2+6 means 2 plus 6 x+y means x plus y
Subtraction:	4-1 means 4 minus 1 x-y means x minus y
Division:	6÷3 means 6 divided by 3 or 6/3 x÷y means x divided by y or x/y
Multiplication:	3•6 or (3) (6) or 3(6) or 3x6 means 3 times 6 x•y or (x)(y) or x(y) or xy means x times y 3•x or (3)(x) or 3(x) or 3x means 3 times x

? = 335.25

To check it, multiply \$335.25 by 4%. It should equal \$13.41, and it does. Or you could have used a ratio.

13.41/? = 4/100 (4% is to 100%)

You could also have worked backwards, taking 4% of each answer until you got \$13.41.

37. The answer is A. For an interest problem like this one, the interest will equal the rate x principal x time. The time is always expressed as some part of a year. In this problem, the interest will equal 8% x 600 x 1/12 (One month is 1/12 of a year).

.08 x 600 x 1/12 = 48 x 1/12 = 48/12 = \$4.

The trick with these types of problems is to remember to express the time in terms of a year. In this case, the 30 days is expressed as one month, or 1/12 of a year.

38. The answer is B. We know the garden is 30 feet by 40 feet. We need to find the perimeter first, to determine how much fencing is needed. Remember, there are four sides. Two will be 40 feet, and two 30 feet. To find the perimeter, we need to add the four sides. 40 + 40 + 30 + 30 = 140 feet. It costs \$1.75 per foot for the fencing. So the cost will be 140 x 1.75 = \$245.

39. The answer is C. Many people miss this question. We need to find what part of her entire year's earnings she earns in April. We know she earns four times as much in April as in each of the other months. One way to do it would be to assign "parts". April would equal 4 parts. Each of the other months would equal one part, so eleven months with one part each would equal eleven parts. The total would then be 15 parts, 4 of which were April's earnings. So April's earnings would equal 4 out of the 15 total parts, or 4/15 Or, you could have assigned imaginary dollar values to see is the relationship more clearly. Imagine she made \$1000 each month. April's earnings would be 4 times that, or \$4000. In the other eleven months she'd make \$11,000. The total would be \$15,000. April's earnings would be \$4000 of the \$15,000, or 4000/15000. It sometimes helps to bring in "real life" examples to help see relationships more clearly.

40. The answer is D. Another ratio problem. A fitting ending. We're told a train travels 70 miles when a bus travels 50 miles. We need to find how many miles the train will travel when the bus travels 60 miles.

$$\frac{\text{train}}{\text{bus}} \text{ as } \frac{\text{train}}{\text{bus}} \qquad \frac{70}{50} \text{ as } \frac{?}{60}$$

One way to do this is to notice that 70 is 7/5 of 50, so the answer will be 7/5 of 60. 7/5 x 60 = 420/5 = 84

ALGEBRAIC EXPRESSIONS

Since we're dealing with both letters and numbers in algebra, we have to be very careful to know exactly what operation we're performing when we use algebraic symbols.

For example, in using 3+x, we're adding 3 and x. If we use 3x, we're multiplying them (see above). We can also add x's: x+x+x=3x. For example, if x stands for the number 5, x+x+x would equal 5+5+5, or 15. 3x would equal 3 times 5, which is also 15. 3+x would equal 3+5, or 8. They could also be negative numbers: -x-x-x=-3x. Both 3x and -3x are examples of terms in algebra. A term is a quantity completely set off from other quantities to the left or right by either a plus or minus sign. In the example, 3x-4yz+8z, the 3x, 4yz, and 8z would all be considered terms (you have to imagine there's a plus sign in front of the 3x, since it's at the beginning of the expression and not a negative number). In the term 3x, the 3 and the x are called factors. Factors are numbers that are multiplied together. The 3x is one term and has two factors: (3) , and (x) . 4yz is one term and has three factors (4), (y), (z). c3z is one term and has three factors (c),(3), and (z). If you had a horrible looking thing like 5a(b-c) you would still have one term. There is a minus sign, but it's inside of a parentheses, which means the result of b-c will be multiplied by 5a, so that the term is still completely set off from other possible quantities (If you are still confused, it will become clearer soon).

The factors that make up a term are called coefficients. In the term 3x, the number 3 is the coefficient of x, and x is the coefficient of 3. 3x and x3 mean the same thing. The numerical coefficient refers to the number in a term (not the letter or letters). In 4ab, 4 is the numerical coefficient. If there is no number in front of a term, then the numerical coefficient should always be understood to be one. y means ly, abc means labc, -x means -1x, -bc means -1bc.

Sometimes the factors which make up a term are the same. For example xxx would mean x is used three times, and would be multiplied by itself 3 times. If x equaled 5, then x would equal 5x5x5. x (which is really 5 in this case), was used as a factor 3 times. aaaa would mean a was used as a factor four times. Because this is rather cumbersome, algebra uses exponents. An exponent is written slightly above and to the right of the factor that is being repeated, and tells how many times the same factor is being repeated.

x^3=xxx, y^4=yyyy 5^3=(5) (5) (5), a^6=aaaaaa. Similarly,$4a^3y$=4aaay,
$6a^3b^4$=6aaabbbb, $-3a^2b$=-3aab, $-12r^2s^3$=-12rrsss.

PLEASE REMEMBER THERE IS A DIFFERENCE BETWEEN EXPONENTS AND NUMERICAL COEFFICIENTS.

Numerical coefficients precede the unknown variables and act as a multiplier. Exponents cause the variable to be multiplied by itself one or more times such as:

$$4a = a+a+a+a = aaaa$$
$$3y = y+y+y = yyy$$

(a^3 would be pronounced "a cubed", or "a to the third power", x^2 would be pronounced "x squared", or "x to the second power", c^6 would be "c to the sixth power", etc. Do not worry if you're not sure what all these a's, b's, c's, and y's represent. They merely represent unknown numbers. There's no way we could know what numbers they represent, because they're not in equations. We're just using them here to illustrate certain algebraic procedures.)

A REVIEW OF SIGNED NUMBERS

Before we continue on, it's best to briefly review positive and negative numbers, called signed numbers, and their operations. Positive numbers are numbers greater than zero. Negative numbers are numbers less than zero.

Addition of signed numbers

The sum of two positive numbers is always positive. Example: +3 plus +8 equals +11.

The sum of two negative numbers is always negative. Example: -7 plus -8 equals -15.

Adding numbers of unlike signs is a little more complicated, and some people have trouble with it. Try adding -24 and +16. If vou're not sure, you could think of money spent as opposed to money earned. If you spent $24, and earned $16, you would have a debt of $8, or -8. WHEN ADDING NUMBERS OF UNLIKE SIGNS, YOU SHOULD FIRST SUBTRACT THE TWO NUMBER. IF THE LARGER NUMBER IS POSITIVE, THE ANSWER WILL BE POSITIVE. IF THE LARGER NUMBER IS NEGATIVE, THE ANSWER WILL BE NEGATIVE.

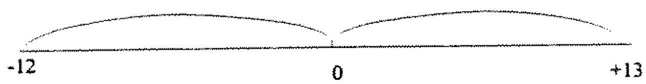
If you're having a problem, try these for practice. (Answers are at end of section).

Add:

1)	(-6) + (2)	3)	(-467) + (421)
2)	(14) + (-16)	4)	(32) + (-13)

When adding more than two numbers, you would add up the like signs first, and then subtract. The sign of the answer will be determined by which sum of positive or negative numbers was larger. Example: (-189) + (52) + (-43) + (112). Adding -189, and -43, we get -232. Adding 52+112 equals 164. Subtracting 164 from 232, we get 68. Because the sum of the negative numbers was greater than the sum of the positive numbers, 68 will be expressed as a negative number, -68.

Subtraction of Signed Numbers



The rule for subtracting two signed numbers is straightforward: TO SUBTRACT TWO SIGNED NUMBERS, ADD THE OPPOSITE OF THE SECOND NUMBER. Another way to express it is to CHANGE THE SIGN OF THE NUMBER BEING SUBTRACTED AND THEN ADD.

Example: (-42) - (-33) would equal -42+33=-9. We changed the sign of the -33, because it was being subtracted, and then added the numbers together to get the answer. Some practice questions follow.

Subtract: (45)-(-23) You would first need to change the sign of the -23 to a +23 and then add the two numbers together. 45+23=68.

If you're having a problem, try these for practice. (Answers are at end of section).:

5) -32-(-28) 6) (16) - (-24)

Example: At the beginning of the month, Alex's checking account had a total of $278.15. At the end of the month, the account was overdrawn by $43.75, excluding bank charges. If there were no deposits during the month, what was the total amount of the checks that Alex wrote?

To answer this, many of us would <u>add</u> the two amounts, $278.15 and $43.75, to get the answer, $321.90. What we could actually be doing here is subtracting the ending balance from the original balance. $278.15-(-43.75) would actually, following the method outlined above, turn into $278.15+ $43.75, as we would have changed the sign of the number being subtracted and then added the numbers together. One more example to illustrate this point.

Example: When you got up at 6 a.m. , the temperature was -12°F. At noon the temperature had risen to +13°F. What was the change in temperature from 6 a.m. to noon?

To find the change, we need to find the difference between the two temperatures, which means subtraction. We need to change the sign of the number being subtracted, and then add the numbers together, 13°-(-12°)= 25°. (If you weren't sure which to subtract from which, it wouldn't have mattered, since all they were asking for was the change in temperature.)

A number line could also illustrate this.

First the temperature had to go up 12° to get from -12° to 0°, then increase an additional 13° to go, from 0° to 13°. Adding this distance, we get 25°.

We've included these examples to try and provide more of a context for these rules about working with positive and negative numbers.

MULTIPLICATION OF TWO SIGNED NUMBERS

When multiplying two numbers: IF THE NUMBERS HAVE THE SAME SIGN, THE PRODUCT IS POSITIVE.

Examples: $(3)(4) = 12$ $(-4)(-6) = 24$

IT'S IMPORTANT TO REMEMBER THAT THE PRODUCT OF TWO NEGATIVE NUMBERS IS POSITIVE.

IF THE NUMBERS HAVE DIFFERENT SIGNS, THE PRODUCT IS NEGATIVE.

Examples: $(-3)(2) = -6$ $(17)(-2) = -34$

If you're having a problem, try these for practice. (Answers are at end of section).:

7) $(-8)(-18)$ 8) $(14)(-7)$

DIVISION OF TWO SIGNED NUMBERS

Division of two positive and two negative numbers is similar to multiplication. THE QUOTIENT (RESULT) OF TWO NEGATIVE NUMBERS IS POSITIVE. THE QUOTIENT OF A POSITIVE AND A NEGATIVE NUMBER IS NEGATIVE.

Example: $(-8) \div (4) = -2$

If you're having a problem, try these for practice. (Answers are at end of section).:

Divide: 9) $(144)\div(-12)$ 10) $(-221)\div(-17)$

MULTIPLYING AND DIVIDING STRINGS OF NUMBERS

TO MULTIPLY OR DIVIDE STRINGS OF NUMBERS, ONE SHOULD FIRST MULTIPLY

AND DIVIDE AS IF THE NUMBERS WERE UNSIGNED. Then, IF THERE IS AN EVEN NUMBER OF MINUS SIGNS, THE RESULT IS POSITIVE. IF THERE IS AN ODD NUMBER OF MINUS SIGNS, THE RESULT IS NEGATIVE.

Example: (5) (-3) (-2) (-6) = -180 There are three minus signs, an uneven number, so the result is negative.

The Order of Algebraic Operations

In algebra, it's necessary to determine the order of operations when we have more than one operation in a problem. For instance, $40 \div 2 - 24 \div 3$. We need to know when to do what, and the following rules should be followed. In an exercise with more than one operation, you should use the following rules: (They apply for arithmetic as well).

1. Do what is inside the parentheses first. (Parentheses are also implied below and above any fraction bar).

2. Do exponents next.

3. Working from left to right do multiplications and divisions as you come to them.

4. Go back to the left and work to the right doing additions and subtractions.

So, it's: Parentheses
Exponents
Multiplication and Division
Addition and Subtraction

In the example, first perform division, then substraction. The problem becomes 20 - 8 = 12.

This is very important to remember. If you're having trouble you can memorize the order by thinking "**P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally." If you want something a little difference you could use "**P**lease **E**xcuse My **D**readful **A**lgebraic **S**kills," or "**P**lease **E**ncourage **M**y **D**aring **A**lgebraic **S**kills," depending upon your outlook.

Answers:

1) -4	6) 40
2) -2	7) 144
3) -46	8) -98
4) 19	9) -12
5) -4	10) 13

GEOMETRY

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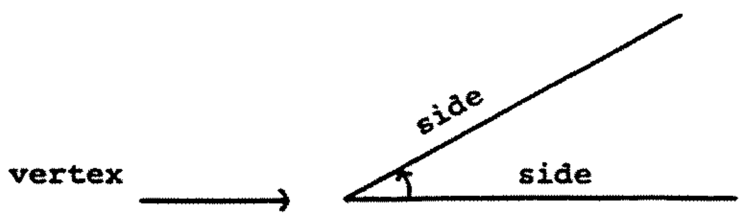
Geometry comes from the Greek words which mean earth and measure. When first used it dealt with measurement of land. Egyptians used geometry to find the boundaries of their farms after the annual floods had washed away the old markers. They later used geometry in the construction of buildings, temples, and pyramids.

Geometry is used to answer questions about the size, shape, volume or position of something. It's professionally used by engineers, architects, navigators, surveyors, carpenters, etc. It has everyday practical applications for someone designing a cabinet, making a pattern for a dress, and deciding how much carpeting to buy for a living room.

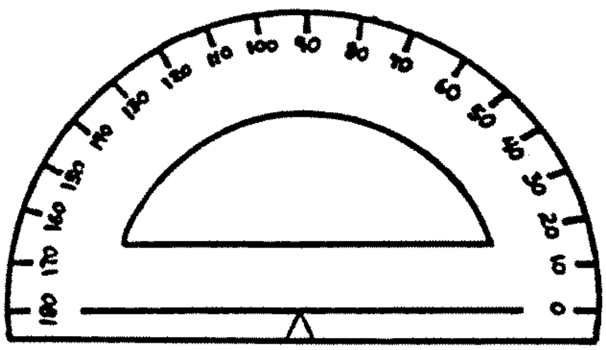
ANGLES

Angles are formed when 2 lines meet a point. The two lines are called SIDES of the angle and the point where they meet is called the VERTEX.

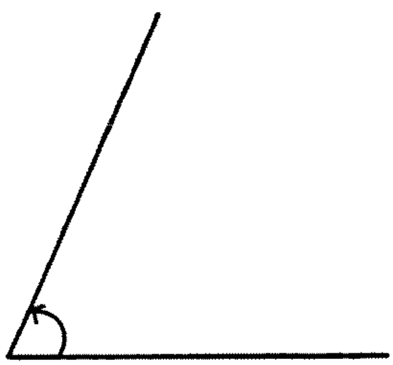


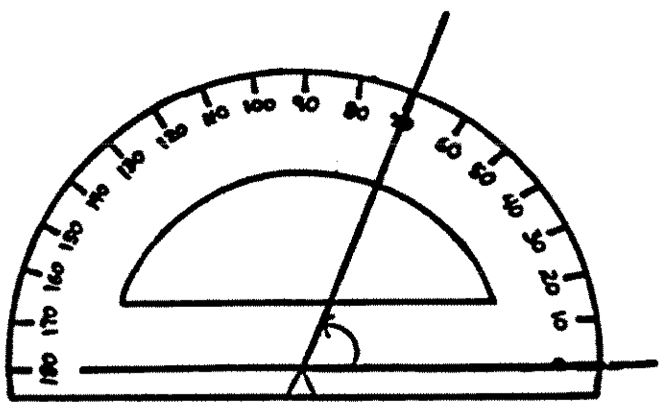
The size of an angle is determined by measuring the amount of openness between the sides. The open area is shown with an arched arrow.

The above angles are progressively larger from left to right as the amount of openness shown by the arched arrow increases. The angle opening is measured in degrees which is symbolized as "°". A 60 degree angle is written as 60°. To measure an angle we use a protractor as shown below.

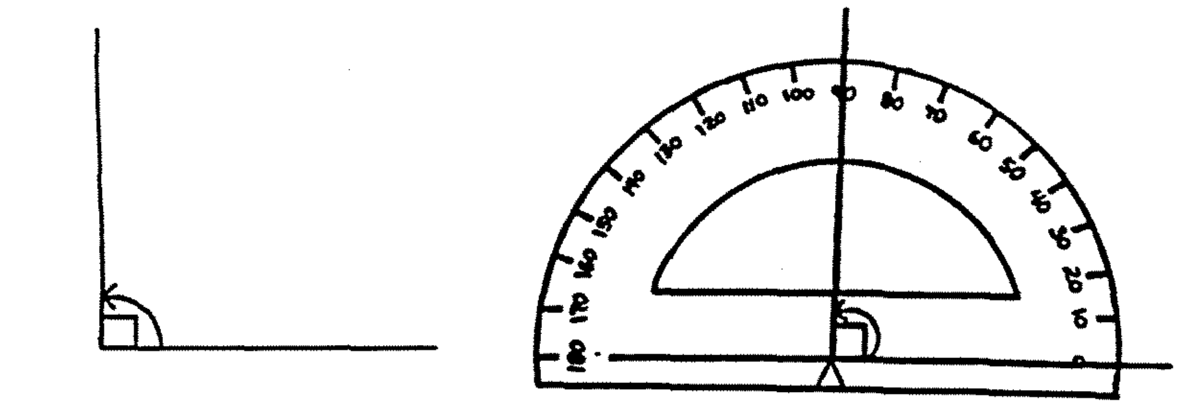


To measure the following angle, place the protractor over the angle so that one side runs along the 0 line.

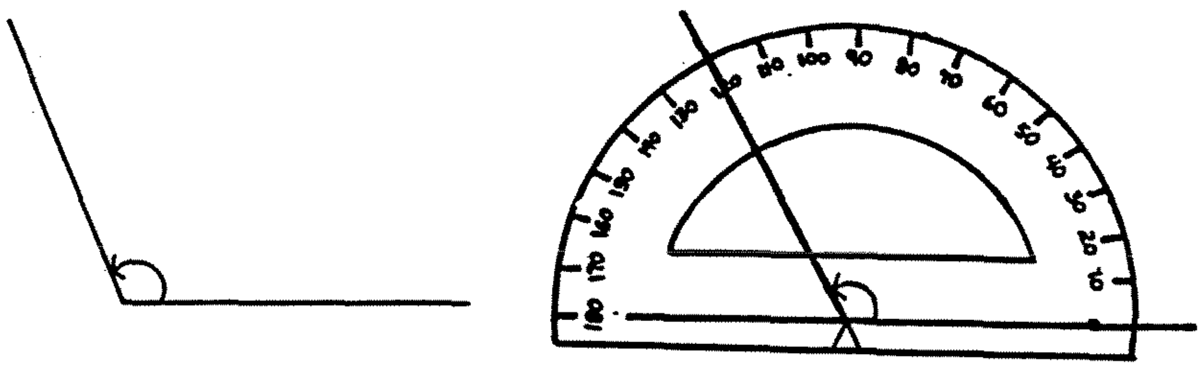




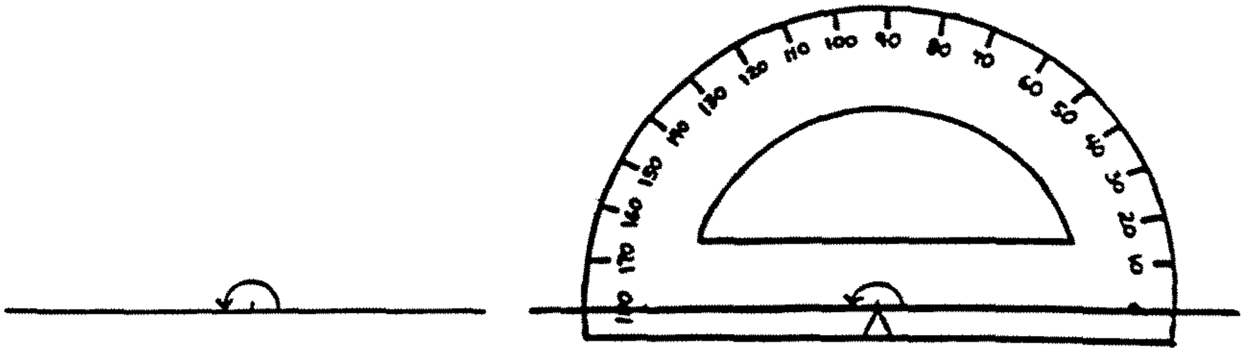
The second side crosses the protractor at 70° so this angle measures 70°. Angles measuring less than 90° are called ACUTE ANGLES.



In the example above the second side crosses the protractor at 90°. Angles measuring 90° are called RIGHT ANGLES. Right angles are shown by placing a small square at the vertex.



In this example the second side crosses the protractor at 120°. Angles measuring more than 90° but less than 180° are OBTUSE ANGLES.



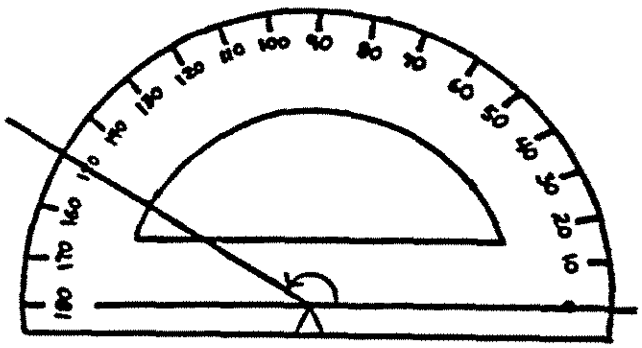
The second side in the example shown above crosses the protractor at 180°. Angles measuring exactly 180° are called STRAIGHT ANGLES. Notice that the arched arrow in a straight angle forms a semi-circle or half a circle. If 180° equals $\frac{1}{2}$ of a circle, then a whole circle must equal 360°, which it does.

Angles measuring more than 180° like those pictured above are REFLEX ANGLES. To determine the number of degrees in a reflex angle, measure the smaller angle and subtract it from 360°.

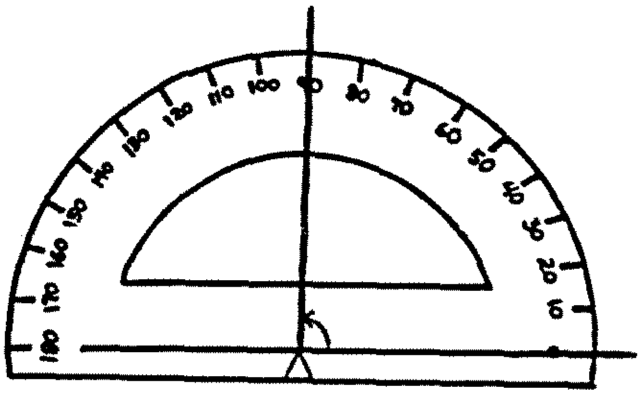
EXERCISE 13

Measure each angle. State how many degrees it has and the letter corresponding to its name:

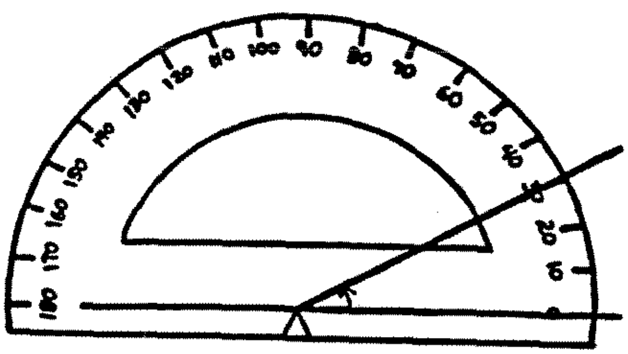
a. acute
b. right
c. obtuse
d. straight
e. reflex



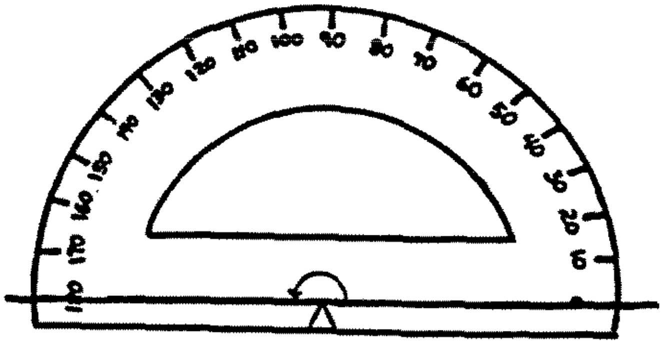
1. ____degrees; ____angle



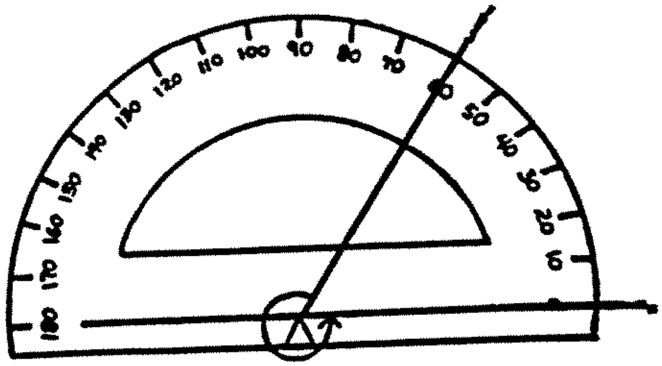
2. ____degrees; ____angle



3. ____degrees; ____angle



4. ____degrees; ____angle



5. ____degrees; ____angle

Match each angle with the letter corresponding to its name.

6. _____125°
7. _____270°
8. _____80°
9. _____180°
10. _____90°

a. acute
b. right
c. obtuse
d. straight
e. reflex

11. A round pizza is divided into 10 equal pieces. Each piece is cut at what angle?.

12. How many degrees are in $\frac{1}{8}$ of a circle?

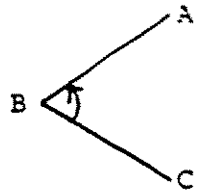
13. A circle is cut in fourths. At what angle is each piece cut?

14. When the big hand on a clock moves from 12 to 6, how many degrees does it pass?

15. When the big hand on a clock moves from 12 to 9, how many degrees does it pass?

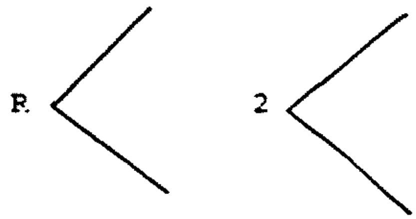
The symbol for angle is "$\angle$". Angles are usually labeled with three letters: one at the end of each side and the third at the vertex.

The angle to the right can be read as $\angle$ABC or $\angle$CBA. It is read as angle ABC or angle CBA. The vertex letter must be in the middle.



Another method of labeling angles is to simply place a letter or number at the angle's vertex.

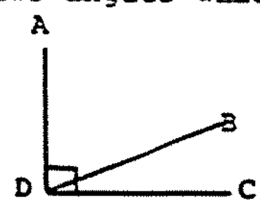
The angles to the right would be written as $\angle$R and $\angle$2 respectively.



PAIRS OF ANGLES

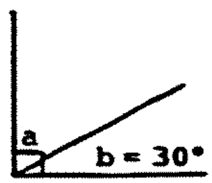
Certain pairs of angles have special relationships and therefore special names. COMPLEMENTARY ANGLES are two angles which total 90°.

If $\angle$ADC is a right angle (90°), then $\angle$ADB and $\angle$BDC are complements of each other.



If you're given the value of one complementary angle and asked to find the other, subtract the known complement from 90° to find the value of the other complement.

In the example to the right, $\angle$a = 90° - 30°, that is 60°. Algebraically this could be stated as follows: If $\angle$a and $\angle$b are complementary, then $\angle$a + $\angle$b = 90°.



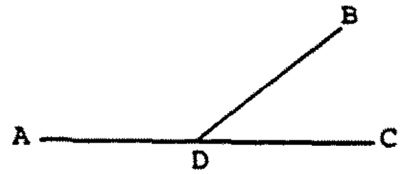
Substituting the known values we get $\quad a + 30° = 90°$

Solving the equation we get... $\quad -30° \quad -30°$

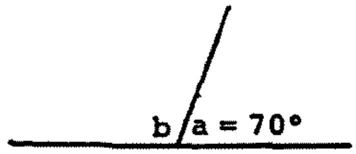
a equals 60 $\quad a = 60°$

SUPPLEMENTARY ANGLES are two angles which total 180°.

If ∠ADC is a straight angle (180°), then ∠ADB and ∠BDC are supplements of each other.



If you are given the value of one supplementary angle and asked to find the other, subtract the known supplement from 180° to find the other supplement.



In the example to the right, ∠b = 180° − 70°, that is 110°. Algebraically this could be stated as follows: If ∠a and ∠b are supplementary, then

$$a + b = 180°$$

Substitute the known values:

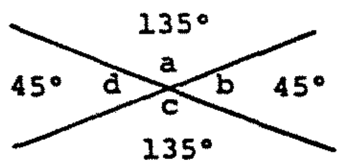
$$b + 70° = 180°$$

Solve for b:

$$\begin{array}{lr} b + 70° & = 180° \\ -70° & -70° \\ \hline b & = 110° \end{array}$$

VERTICAL ANGLES are formed when two straight lines cross. The vertical angles are opposite each other and are equal to each other.

In the figure to the right, ∠a is vertical to ∠c, and ∠b is vertical to ∠d. If a = 135°, then ∠c must also equal 135°. If ∠b equals 45°, then ∠d must also equal 45°.



Some questions in regard to pairs of angles are best solved algebraically.

EXAMPLE: Angles a and b are supplementary. Angle b is 4 times ∠a. What are the two angles?

Let ∠a = a
Let ∠b = 4a

The two angles must total 180° so an equation can be written and solved as follows:

$a + 4a = 180°$
$5a = 180°$
$a = 36°$

∠a = 36° and ∠b, which equals 4a or 4(36), equals 144°

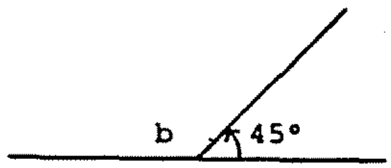
EXERCISE 14

1. If $\measuredangle 1$ and $\measuredangle 2$ are complementary, and $\measuredangle 1$ is 65°, what is $\measuredangle 2$?

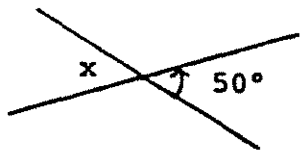
2. Angles x and y are supplementary. If angle x = 50°, what is angle y?

3. Angle m and angle p are vertical angles. If angle m = 45°, what does angle p equal?

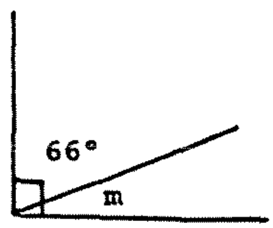
4. In the figure to the right, write an algebraic equation to find $\measuredangle b$, then solve for b.



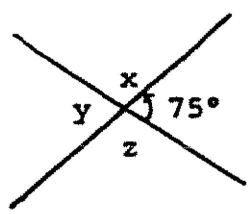
5. In the figure to the right, what is $\measuredangle x$?



6. In the figure to the right, write an algebraic equation and solution for $\measuredangle m$.



7. In the figure to the right, what are the values of angles x, y and z?



8. Two angles are supplementary. One angle is twice the other plus 30°. What are the two angles?

9. Two angles are complementary. If one angle is twice the other, what are the two angles?

10. When the big hand on a clock moves from 12 back to 12 (one complete revolution), how many degrees does it pass through?

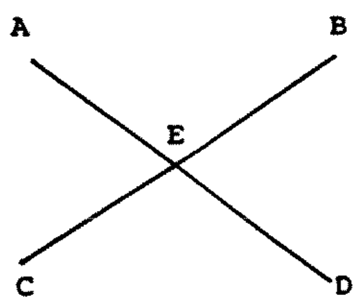
11. When the small hand on a clock moves from 1 to 2, how many degrees does it pass through?

12. The big hand on a clock is on 11 and the small hand is on 3. The obtuse angle they form has how many degrees?

13. The angles formed by the Red Cross each have how many degrees?

14. A ladder leaning against a house forms a 72° angle with the ground. What is the size of the supplementary angle formed with the ground?

15. In the figure to the right, what angle is vertical to ∡AEC?

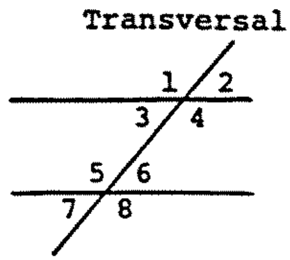


TRANSVERSAL PASSING THROUGH PARALLEL LINES

Two straight lines running side-by-side and never meeting are called PARALLEL LINES. The symbol for parallel is "//".

In the figure to the right, line AB is parallel to line CD. Symbolically this is stated as AB//CD.

A straight line passing through parallel lines is called a **TRANSVERSAL.** The 8 angles formed when a transversal passes through parallel lines have special relationships.



Study the figure above. Notice that if the top set of angles (angles 1, 2, 3 and 4) were picked up and placed over the bottom set of angles (5, 6, 7 and 8), there would be a perfect fit.

Therefore: $\angle 1 = \angle 5$
$\angle 2 = \angle 6$
$\angle 3 = \angle 7$
$\angle 4 = \angle 8$

These four pairs of angles are said to be CORRESPONDING ANGLES. Corresponding angles are equal.

ALTERNATE INTERIOR ANGLES are also equal. Alternate interior angles are on the inside of the parallel lines and opposite each other:

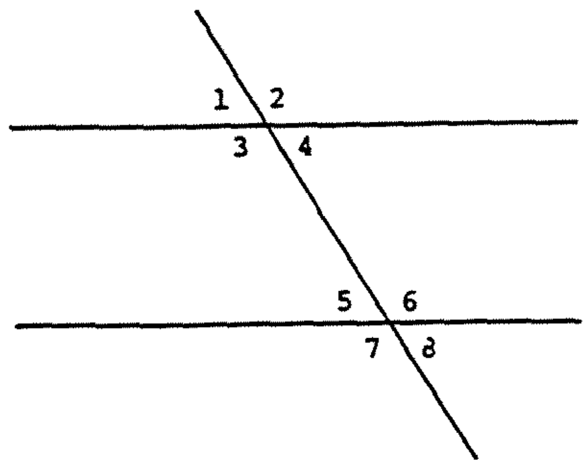
$\angle 3$ and $\angle 6$ are alternate interior angles
$\angle 4$ and $\angle 5$ are alternate interior angles

Finally, ALTERNATE EXTERIOR ANGLES are equal. Alternate exterior angles are outside of the parallel lines and opposite each other:

$\angle 1$ and $\angle 8$ are alternate exterior angles
$\angle 2$ and $\angle 7$ are alternate exterior angles

Given parallel lines intersected by a transversal and the value of one angle, you can determine the other seven angles.

EXAMPLE:



Given $\measuredangle 1 = 60°$,

$\measuredangle 2 = 120°$ because it's angle 1's supplement

$\measuredangle 3 = 120°$ because it's vertical to $\measuredangle 2$

$\measuredangle 4 = 60°$ because it's vertical to $\measuredangle 1$

$\measuredangle 5 = 60°$ because it corresponds to $\measuredangle 1$

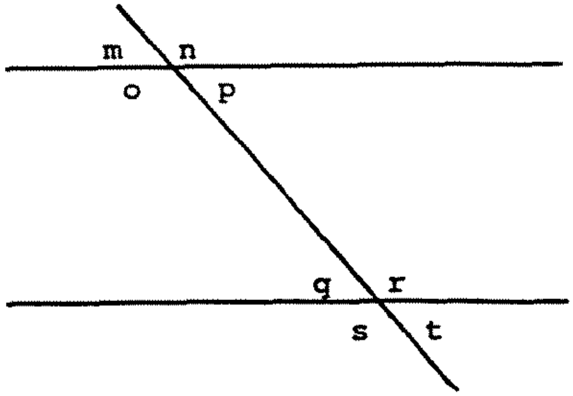
$\measuredangle 6 = 120°$ because it corresponds to $\measuredangle 2$

$\measuredangle 7 = 120°$ because it corresponds to $\measuredangle 3$

$\measuredangle 8 = 60°$ because it corresponds to $\measuredangle 4$

EXERCISE 15

USE the figure below to answer questions 1-5.



1. What angle corresponds to ∡o? S

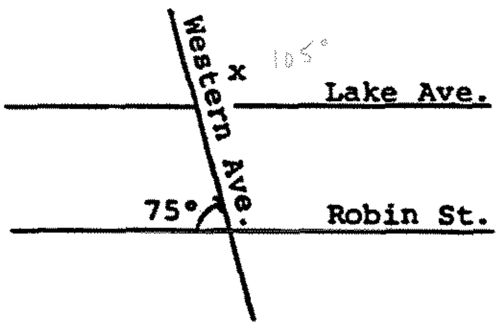
2. What angle is an alternate exterior to ∡n? s

3. What angle is the alternate interior to ∡p? q

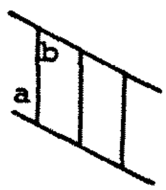
4. If ∡m = 50°, what does ∡s equal? 130°

5. If ∡o = 130°, what does ∡t equal? 50°

6. Western Avenue crosses Lake Avenue and Robin Street at an angle of 75° as shown below. What is the value of ∡x?



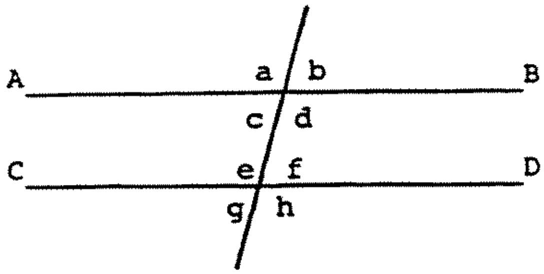
7. If ∡a = 65° in the hand railing shown to the right, what is ∡b? 65°



8. In the letter A pictured to the right, what is the value of ∡b?

180
-140
40°

Use the figure below to answer questions 9-15. AB//CD; a = 105°.



9. What is ∡b?

10. What is ∡c?

11. What is ∡d?

12. What is ∡e?

13. What is ∡f?

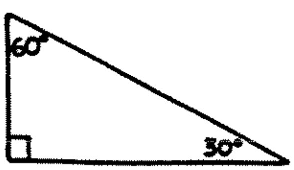
14. What is ∡g?

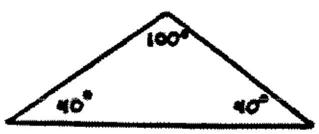
15. What is ∡h?

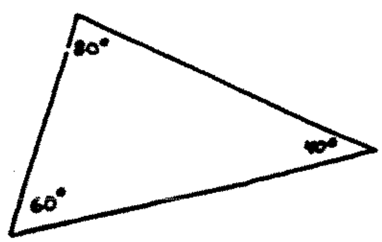
TRIANGLES

A TRIANGLE is a plane (flat) figure with 3 sides and 3 angles. The symbol for triangle is "Δ". The 3 angles of a triangle always add up to 180° no matter what the shape of the triangle.

EXAMPLES:



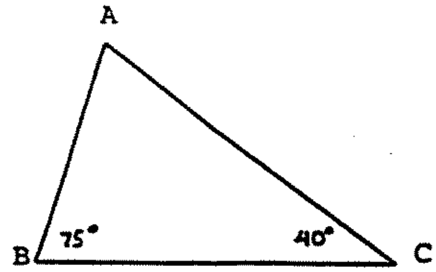




Given the values of 2 angles in a triangle, the third can be determined by subtracting the sum of the given angles from 180°. The relationship can be stated and solved algebraically as:

$$\angle 1 + \angle 2 + \angle 3 = 180°$$

EXAMPLE:



$$\angle A + \angle B + \angle C = 180°$$

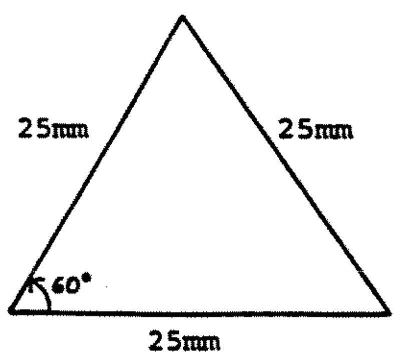
$A + 75° + 40° = 180°$ (substituting values)

$A + 115° = 180°$

$A = 65°$

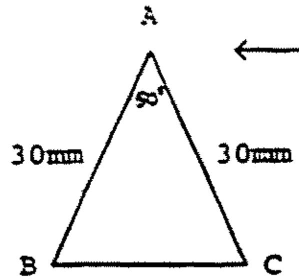
The shape of a triangle determines its name. Knowing the shapes and properties of the different kinds of triangles will enable you to solve certain kinds of problems dealing with triangles.

An EQUILATERAL TRIANGLE has 3 equal angles, each measuring 60°. It also has 3 equal sides.



An ISOSCELES TRIANGLE has two equal angles which are called the BASE ANGLES. The sides opposite the base angles are also equal. The third angle in an isosceles triangle is called the VERTEX ANGLE.

vertex angle



base angle → B C ← base angle

In the isosceles triangle above, how many degrees are in each base angle?

$$\angle A + \angle B + \angle C = 180°$$

$$50° + B + C = 180°$$

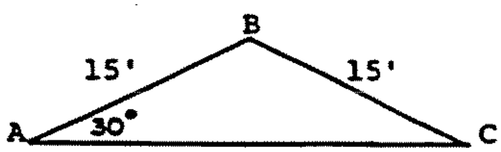
$$B + C = 130°$$

Since $\angle B$ and $\angle C$ are equal, we can divide 130° by 2 to find what each is equal to:

$$130 \div 2 = 65°$$

$$\angle B \text{ and } \angle C \text{ each} = 65°$$

What is the value of $\angle B$ and $\angle C$ in the triangle shown below?



1. Because sides AB and BC are equal, the opposite angles ($\angle A$ and $\angle C$) must also be equal. Therefore, $\angle C$ must equal 30°.

2. The vertex angle is found by substituting known values.

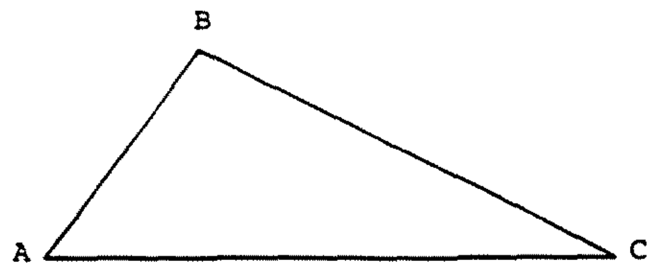
$$\angle A + \angle B + \angle C = 180°$$

$$30° + B + 30° = 180°$$

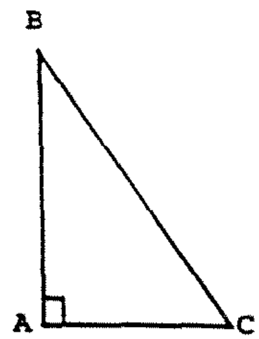
$$B + 60° = 180°$$

$$B = 120°$$

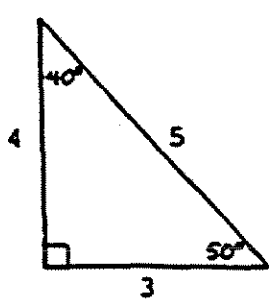
A SCALENE TRIANGLE has no equal sides and no equal angles. It is the most common of triangles.



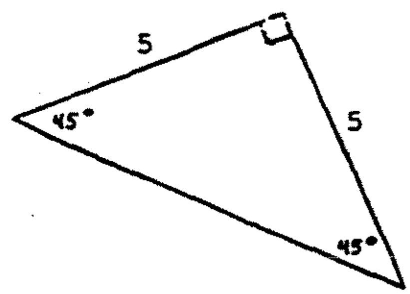
A RIGHT TRIANGLE has one right angle. The side across from the right angle is called the HYPOTENUSE. The other two sides are called LEGS. In the example shown below, $\measuredangle A$ is a right angle, side BC is the hypotenuse, and sides AB and AC are legs.



A right triangle could also be an isosceles triangle or a scalene triangle as shown below.



scalene right triangle

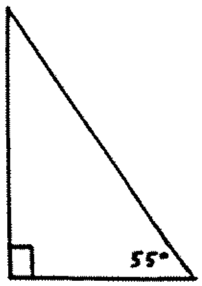


isosceles right triangle

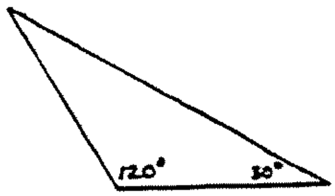
EXERCISE 16A

Find the value of each unmeasured angle in the triangles below.

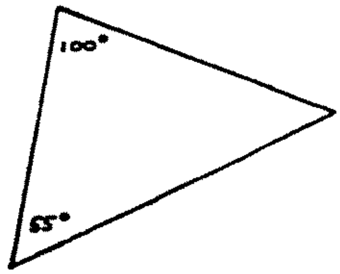
1.



2.



3.



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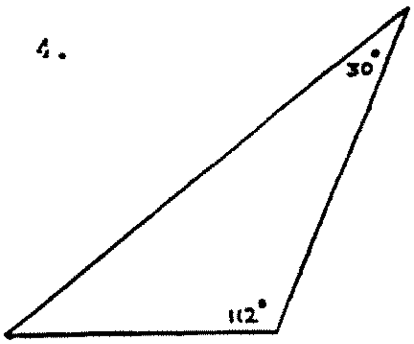
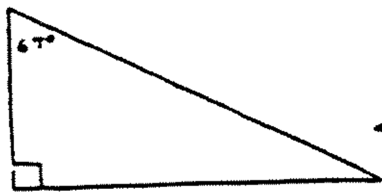
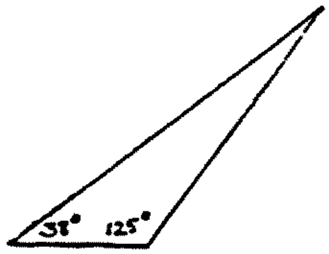


5.



6.



7. In triangle RJK, ∡K is 80° and ∡J is 20°. What is the value of ∡R?

8. In triangle ABC, ∡A is 40° and ∡B is a right angle. What is the value of ∡C?

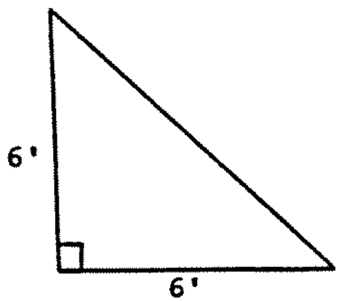
9. An isosceles triangle has a base angle of 35°. What is the vertex angle?

10. Triangle APD is equilateral. What is the value of ∡A?

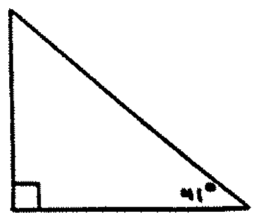
For questions 11-16, write:

(1) the value of the unnamed angle(s)
(2) the name of the triangle (equilateral, isosceles, or scalene)

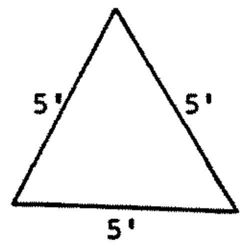
11.



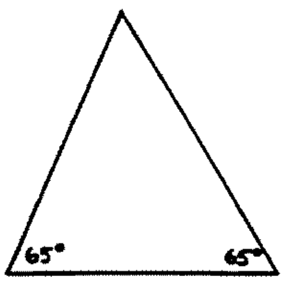
12.



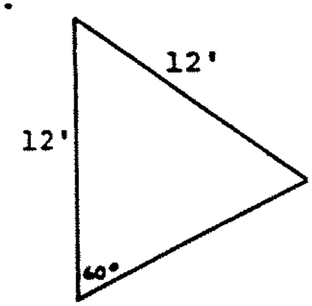
13.



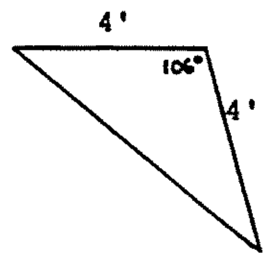
14.



15.



16.



17. The base angle of an isosceles triangle is 21°. What is the vertex angle?

18. The side of an isosceles triangle opposite a base angle is 6'. What is the value of the side opposite the other base angle?

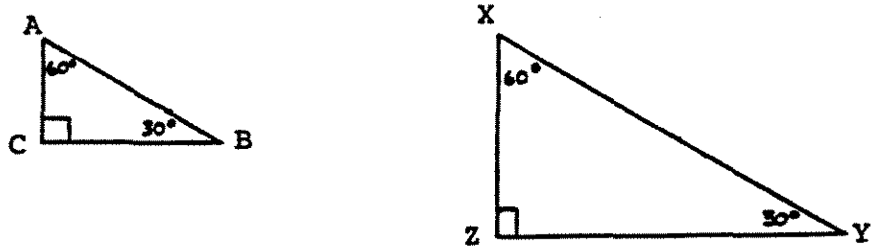
19. In triangle ABC, $\measuredangle A = 60°$ and $\measuredangle B = 60°$. Side AB = 40 feet. What does side BC measure?

20. In a scalene right triangle, one angle is 28°. What is the value of the other acute angle?

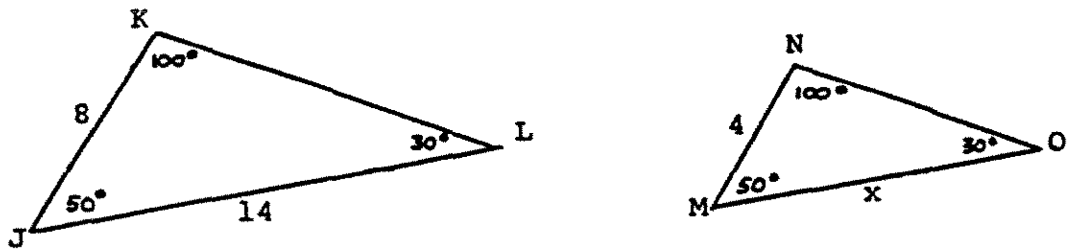
SIMILAR TRIANGLES

Two triangles that have the same angles are called SIMILAR TRIANGLES. Similar triangles have the same shape but the length of their sides are different.

In the example shown below, triangle ABC is similar to triangle XYZ. The symbol for "is similar to" is "$\sim$". The relationship of the two triangles is expressed as $\triangle ABC \sim \triangle XYZ$.



Given the similar triangles shown below, it is possible to determine side MO by setting up a proportion.



STEP 1: Identify corresponding sides. Corresponding sides are opposite the equal angles. JK is to MN as JL is to MO. In fraction form, this proportion is stated as:

$$\frac{JK}{MN} = \frac{JL}{MO}$$

STEP 2: Substitute known values for the sides represented in the proportion:

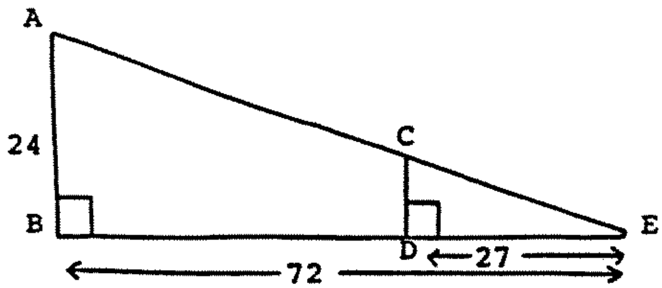
$$\frac{8}{4} = \frac{14}{x}$$

STEP 3: Solve the proportion for the unknown:

$$8x = 56$$
$$x = 7$$

Similar triangles are the basis of many popular test questions and practical applications.

EXAMPLE 1: In the triangle shown below, AB = 24, BE = 72, and DE = 27. What is the length of CD?



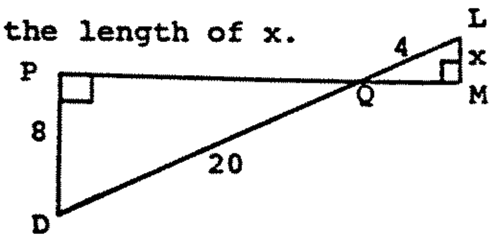
Triangle ABE $\sim$ triangle CDE because both share $\measuredangle$E and each has a right angle, which means the third angles ($\measuredangle$BAE and $\measuredangle$DCE) must also be equal.

$\frac{AB}{CD} = \frac{BE}{DE}$ 1. Setting up proportion.

$\frac{24}{q} = \frac{72}{27}$ 2. Substituting known values.

$72q = 648$
$q = 9$ 3. Solving the equation.

EXAMPLE 2: Find the length of x.



Triangle PDQ $\sim$ triangle LMQ because both have right angles and equal vertical angles ($\measuredangle$PQD and $\measuredangle$LQM), so their third angle must also be equal.

$$\frac{PD}{LM} = \frac{DQ}{QL}$$

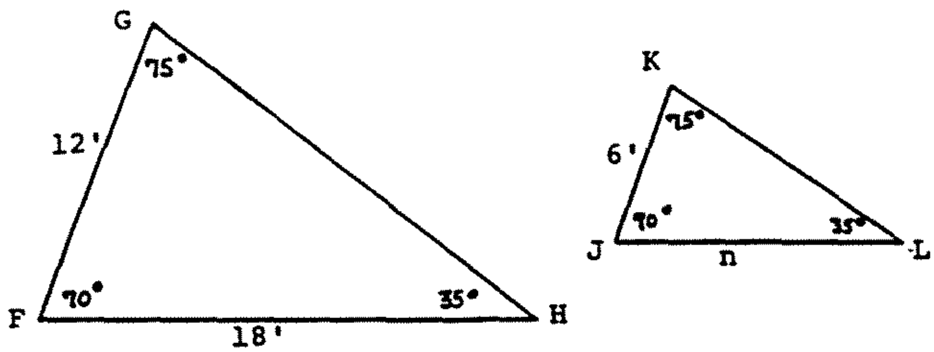
$$\frac{8}{x} = \frac{20}{4}$$

$$20x = 32$$

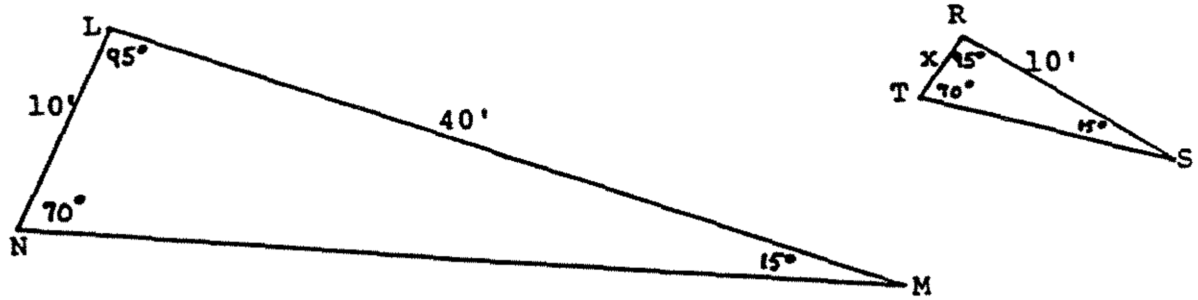
$$x = \frac{32}{20} = 1\frac{3}{5}$$

EXERCISE 16B

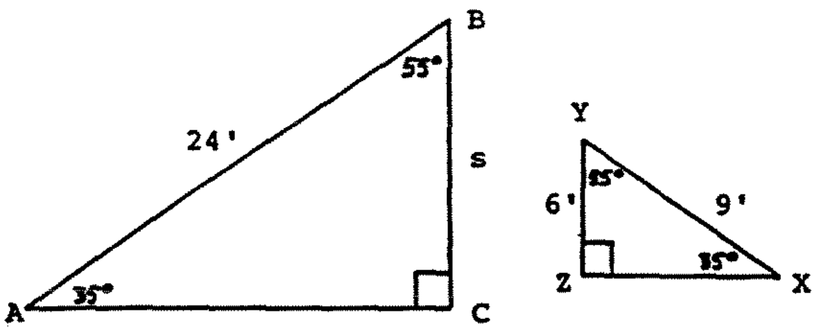
1. $\triangle$ FGH $\sim$ $\triangle$ JKL. What is the length of side n?



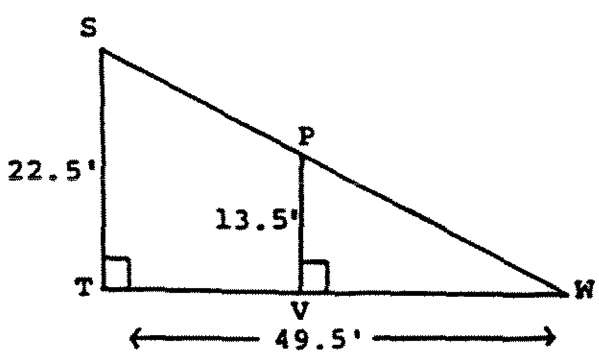
2. $\triangle$ LMN $\sim$ $\triangle$ RST. What is the length of side x?



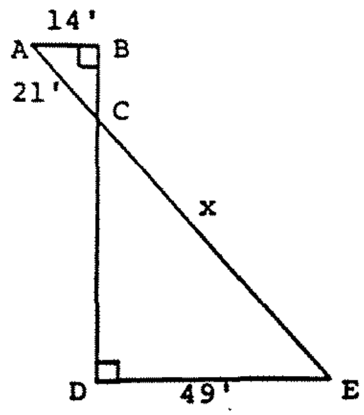
3. $\triangle$ ABC $\sim$ $\triangle$ XYZ. What is the length of side s?



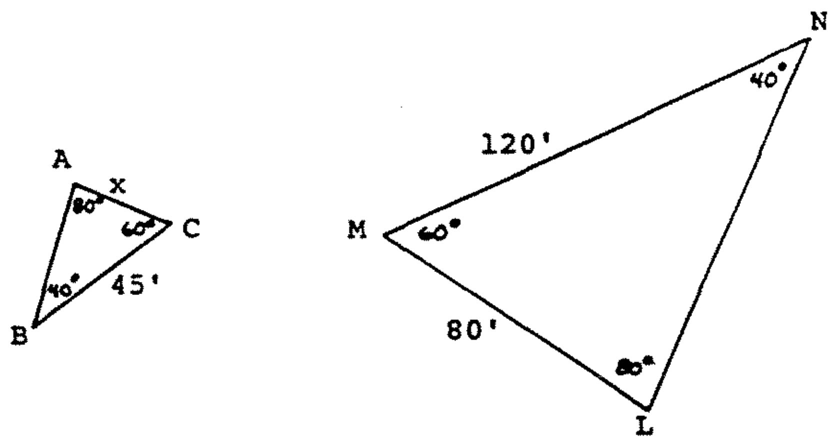
4. ST = 22.5, PV = 13.5, and TW = 49.5. What is the length of VW?



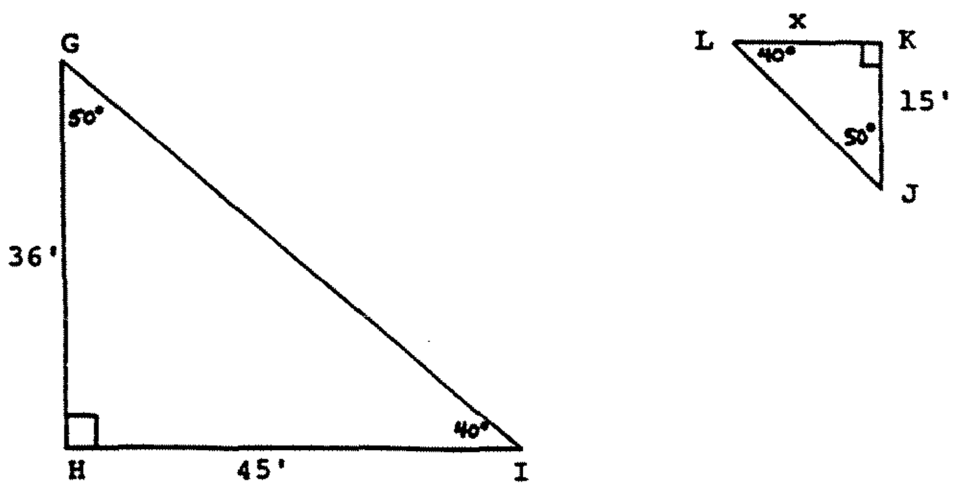
5. AB = 14, AC = 21, and DE = 49. What is the value of CE?



6. Given the figures shown below, find side x.

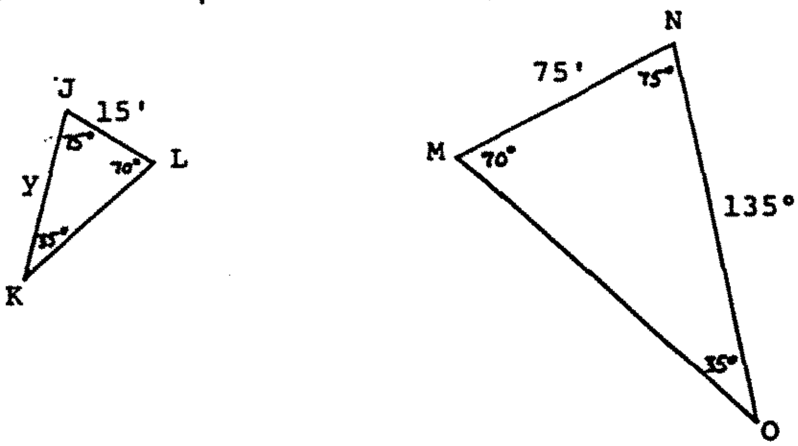


7. Given the figures shown below, find side x.

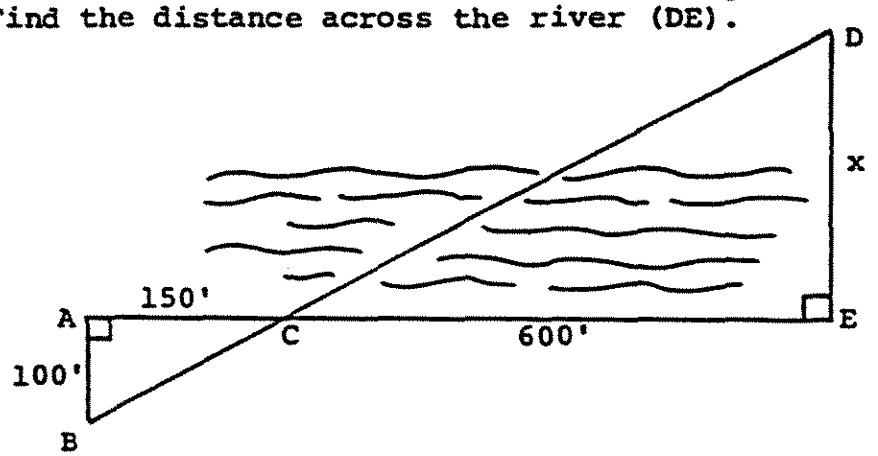


8. Find the value of side y in the figures shown below.

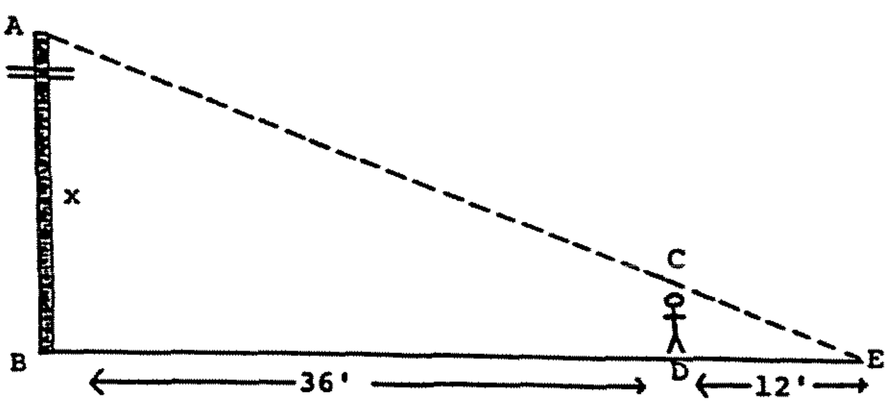
NOTE: $\measuredangle J = 75°$, $\measuredangle K = 35°$, and $\measuredangle L = 70°$.



9. To find the distance across the Hudson River, Tom measured the distances shown in the diagram below. Find the distance across the river (DE).



10. To measure the height of a phone pole, Maria, who is 5 feet tall, measured her shadow and the shadow cast by the pole as shown in the diagram below. How tall is the phone pole?

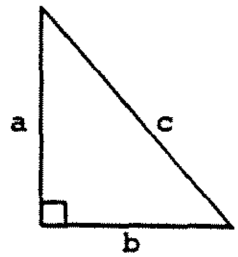


PYTHAGOREAN THEOREM

The Greek mathematician, Pythagoras, long ago discovered the relationship of the hypotenuse of a right triangle and the other two sides. He found that the square of the hypotenuse was equal to the sum of the squares of the other two sides. The relationship is known as the PYTHAGOREAN THEOREM.

As it relates to the triangle on the right, it is symbolically represented as:

$$c^2 = a^2 + b^2$$

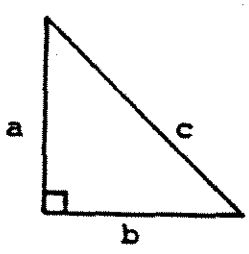


If any two sides of a right triangle are known, the third can be found by substituting the known values in the formula.

EXAMPLE 1: In the triangle represented above, if a = 4 and b = 3, find the value of c.

$$c^2 = a^2 + b^2$$
$$c^2 = 4^2 + 3^2$$
$$c^2 = 16 + 9$$
$$c^2 = 25$$
$$c = \sqrt{25}$$
$$c = 5$$

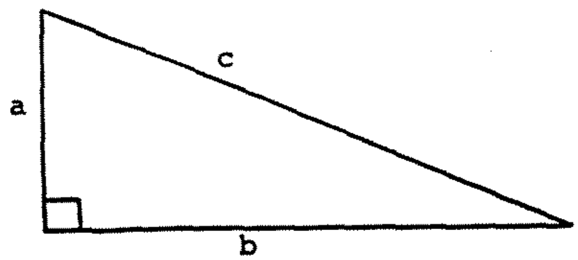
EXAMPLE 2: In the right triangle shown below, side c = 10 and side b = 6. What is the value of side a?



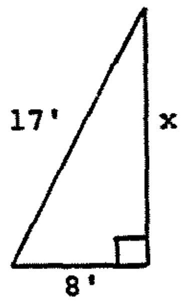
$$c^2 = a^2 + b^2$$
$$10^2 = a^2 + 6^2$$
$$100 = a^2 + 36$$
$$a^2 = 64$$
$$a = \sqrt{64}$$
$$a = 8$$

EXERCISE 16C

1. In the triangle shown below, find side c, if side a = 5' and side b = 12'.

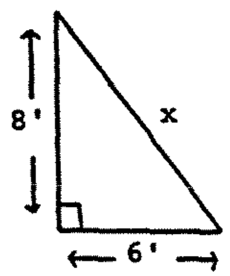


2. In the triangle shown below, find the length of the unknown leg (x).

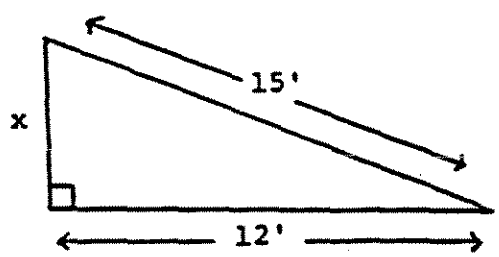


3. In triangle XYZ, the hypotenuse (YZ) is 13 feet long. Side XY is 5 feet long. What is the length of side XZ?

4. In the right angle shown below, find the length of the hypotenuse.



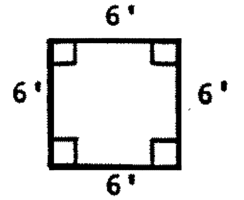
5. Find the length of the missing leg in the triangle shown below.



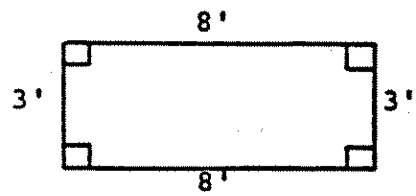
QUADRILATERALS

QUADRILATERALS are plane (flat) figures with four sides. The interior angles of all quadrilaterals total 360°. There are five (5) special types of quadrilaterals. Knowing their characteristics will help in solving many types of problems.

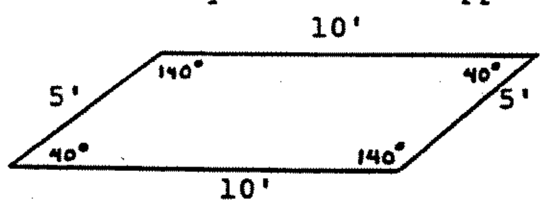
A SQUARE has 4 equal sides, 2 pairs of parallel sides, and 4 right angles.



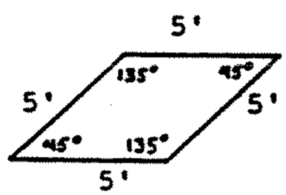
A RECTANGLE has 4 sides with 2 pairs of parallel sides and 4 right angles. The opposite sides of rectangles have equal sides.



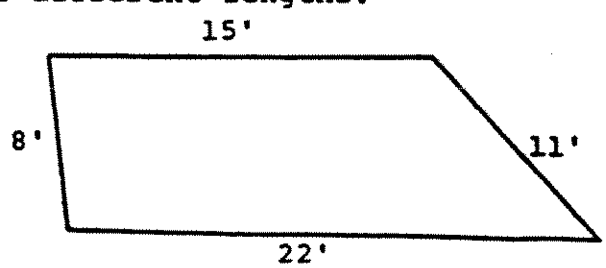
A PARALLELOGRAM has 4 sides with 2 pairs of parallel sides. The opposite sides are equal and the opposite angles are equal.



A RHOMBUS has 4 equal sides, 2 pairs of parallel sides, and 2 pairs of equal angles.

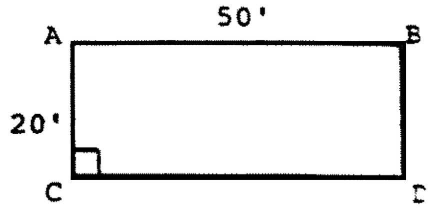


A TRAPEZOID has four sides. Two sides are parallel. The parallel sides are called BASES. All four sides of a trapezoid may be of different lengths.



EXERCISE 17

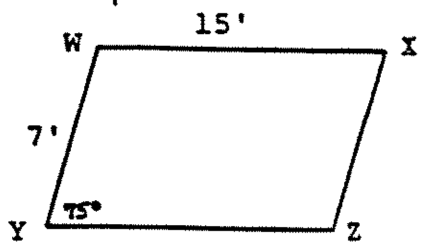
1. In rectangle ABCD, side AB = 50' and side AC = 20'. What is the length of side BD?



2. In rectangle ABCD, what is the length of side CD?

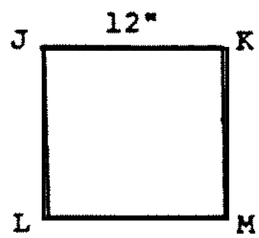
3. In rectangle ABCD, how many degrees are in $\angle$A?

4. In parallelogram WXYZ, side WX = 15', side WY = 7', and $\angle$Y = 75°. What is the length of side XZ?



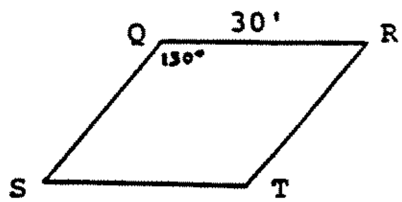
5. In parallelogram WXYZ, how many degrees are in $\angle$Z?

6. In square JKLM, side JK is 12". What is the length of side KM?



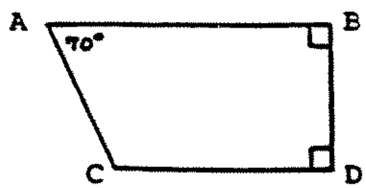
7. In square JKLM, how many degrees are there in $\angle$L?

8. In rhombus QRST, side QR = 30' and $\angle$Q = 130°. What is the length of side QS?



9. In rhombus QRST, how many degrees are in $\angle$T?

10. In the figure to the right, $\angle$B and $\angle$D are right angles, $\angle$A = 70°, and AB//CD. What is the figure called?



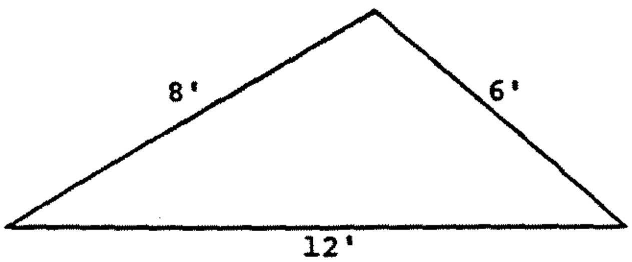
11. What are AB and CD called in problem 10?

12. How many degrees are there in $\angle$C in problem 10?

PERIMETER

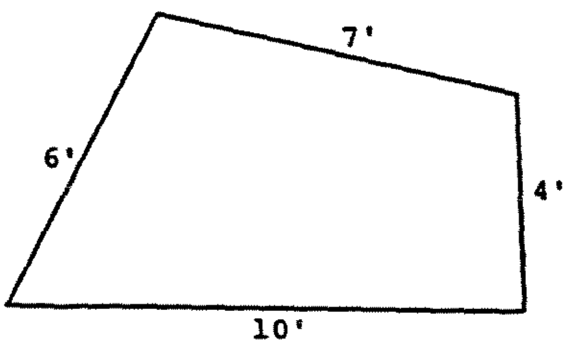
The distance around any closed, plane (flat) figure is the PERIMETER. To find the perimeter of a geometric figure having sides, add the length of each side.

EXAMPLE 1: Find the perimeter of the triangle shown below.



The perimeter = the sum of the sides
= 8' + 6' + 12'
= 26'

EXAMPLE 2: Find the perimeter of the quadrilateral shown below.



The perimeter = the sum of the sides
= 6' + 7' + 4' + 10'
= 27'

Knowing the properties of geometric figures discussed so far will enable you to find their perimeters even though the length of each side isn't given.

EXAMPLE 3: Find the perimeter of a square that is 12' long on one side. By definition a square has four equal sides. Therefore if one side is 12' long, all four sides must be 12' long. The perimeter of a square can be expressed algebraically as:

perimeter = 4(one side)

OR

$$p = 4s$$

In example 3 we can substitute 12 for s to get

$$p = 4(12)$$
$$p = 48'$$

EXAMPLE 4: A rectangle measures 10' by 12'. What is the perimeter? By definition a rectangle has two pairs of equal, parallel sides. Therefore if one side is 10', its parallel side is also 10'. If the other side is 12', its parallel side is also 12'. An algebraic formula for the perimeter of a rectangle can be written as follows:

perimeter = 2 times the length + 2 times the width

OR

$$p = 2(l) + 2(w)$$

Substituting the values of length and width in example 4 we get

$$p = 2(12) + 2(10)$$
$$p = 24 + 20$$
$$p = 44'$$

EXERCISE 18

1. One side of a parallelogram is 15 feet and the other is 10 feet. What is its perimeter?

2. One side of a rhombus is 25". What is its perimeter?

3. One side of an equilateral triangle is 15 feet. What is its perimeter?

4. What is the perimeter of a rectangle that measures 15 feet by 20 feet?

5. One of the two equal sides of an isosceles triangle measures 16 meters and the third side measures 10 meters. What is the triangle's perimeter?

6. The bases of a trapezoid measure 20' and 16' respectively. The other two sides measure 4' and 6'. What is the perimeter of the trapezoid?

7. The legs of a right triangle measure 6' and 8'. What is the perimeter of the triangle? (Hint: First find the length of the hypotenuse.)

8. Maria's garden measures 20' by 30'. How many feet of fence does she need to enclose the garden?

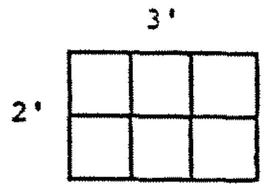
9. A window measures 72" by 32.5". How many inches of weather stripping are needed to insulate the window?

10. A baseball diamond is actually a square measuring 90' on a side. How many feet must a player run in order to score? (To score a player must run around the "diamond".)

AREA

The amount of surface on a plane (flat) figure is its AREA. Area is measured in square inches, square feet, square yards, etc.

The area of the rectangle to the right is the total number of one-foot squares that make up its surface -- 6 square feet.



The formula for the area of a rectangle is A = lw, that is:

$$\text{Area} = \text{length times width}$$

EXAMPLE: Find the area of a rectangle measuring 12' by 10'.

$$A = lw$$
$$A = 12(10)$$
$$A = 120 \text{ square feet (sq. ft.)}$$

Given the area and length of a rectangle, the width can be found by substituting the known values in the formula.

EXAMPLE: What is the width of a rectangle which measures 20 feet in length and has an area of 320 sq. ft.?

$$A = lw$$
$$320 = 20w$$
$$W = 16'$$

The formula for the area of a square is given as: Area is the square root of a side or:

$$A = s^2$$

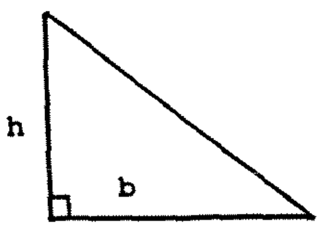
EXAMPLE: Find the area of a square that measures 30" on one side.

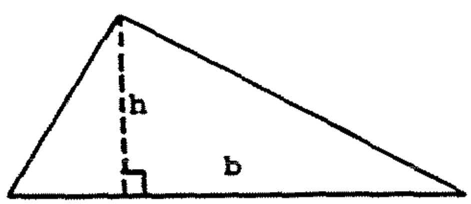
$$A = s^2$$
$$A = 30^2$$
$$A = 900 \text{ sq. in.}$$

The formula for the area of a triangle is given as: $\frac{1}{2}$ the base times the height or:

$$A = \frac{1}{2}bh$$

The height of a triangle is a line that is PERPENDICULAR to the base. A line perpendicular to the base forms right angles with the base. Study the examples below (h = height, b = base).





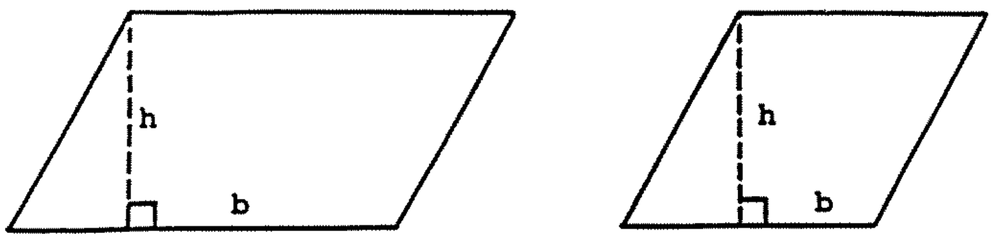
EXAMPLE: Find the area of a triangle having a base of 47' and a height of 20'.

$$A = \frac{1}{2}bh$$
$$A = \frac{1}{2}(47)(20)$$
$$A = 470 \text{ sq. ft.}$$

The formula for the area of a parallelogram or rhombus is given as: Area = base times height or:

$$A = bh$$

Study the examples below.



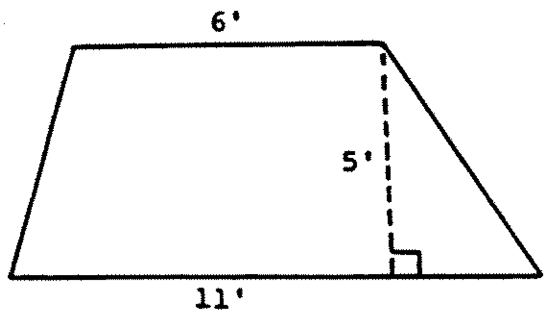
EXAMPLE: Find the area of a parallelogram having a base of 40 feet and a height of 30 feet.

$$A = bh$$
$$A = 40(30)$$
$$A = 1{,}200 \text{ sq. ft.}$$

The formula for the area of a trapezoid is given as: Area = $\frac{1}{2}$ the sum of the two bases times the height. That is:

$$A = \frac{1}{2}(b_1 + b_2)h$$

EXAMPLE: In the trapezoid below, find the area.



$$A = \frac{1}{2}(b_1 + b_2)h$$

$$A = \frac{1}{2}(6 + 11)5$$

$$A = \frac{1}{2}(17)5$$

$$A = 42.5 \text{ sq. ft.}$$

EXERCISE 19

1. What is the area of a yard measuring 60' by 40'?

2. The area of a triangle is 1,280 sq. ft. If the base is 16 feet, what is the height?

3. Find the area of a trapezoid that has bases of 15 meters and 25 meters, and a height of 20 meters.

4. Felipa wants to buy a carpet for her living room which measures 12' by 21'. How many square yards of carpeting will she need? (Hint: Change feet to yards before multiplying).

5. What is the area of an isosceles triangle that has a base of 20 feet, height of 16 feet, and equal sides of 18 feet?

6. A parallelogram has a height of 36" and a base of 40". What is its area?

7. Building costs at the Cape are about $80/sq. ft. of living space. The downstairs living space of a home that Annie and John want to build measures 30' by 40' and the upstairs living space is 30' by 20'. What will it cost to construct the house?

8. How many square feet are in a square yard?

9. How many square inches are in a square foot?

10. A right triangle has legs measuring 15" and 11". What is its area?

11. A cabbage patch measures 10 yards by 24 yards. If a pound of fertilizer will cover 12 square yards, how many pounds will be needed for the cabbage patch?

12. Fritz plans to paper 3 walls each measuring 20' by 10'. If one roll of paper covers 30 sq. ft., how many rolls will he need.

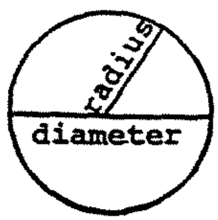
13. What is the area of a rhombus having a height of 12" and a side of 10"?

14. A triangle measures $2\frac{1}{2}$ feet in height and has a base of $1\frac{1}{5}$ feet. What is its area?

15. What is the area of a square measuring 10.5" on a side?

CIRCLES

A CIRCLE is a curved, closed, plane (flat) geometric figure. Each point on the curve is an equal distance from the center. The distance from the center to any point on the curve is called the RADIUS. The DIAMETER of a circle is a line passing through the circle's center as shown below.



The distance around a circle is its CIRCUMFERENCE. The circumference of a circle could be thought of as the circle's perimeter.

To find the circumference of a circle, a special number represented by the symbol "π" (pi) is used. π is a Greek letter having a value of 3.14 or $3\frac{1}{7}$ which is usually expressed as the improper fraction $\frac{22}{7}$. (Depending on which one you use, you may get slightly different answers. In the exercises that follow, we indicate which number you should use to get exactly the same answers as we have.) The formula for the circumference of a circle is π times the diameter. That is:

$$C = \pi d$$

EXERCISE 20 (Use $\frac{22}{7}$ for π unless otherwise noted.)

1. Find the circumference of a circle having a diameter of 14 meters.

2. What is the area of a circle with a diameter of 40 inches? Use 3.14 for π. (Note: The radius is $\frac{1}{2}$ the diameter.)

3. Cynthia needs to fence in a corral having a diameter of 50 feet. How many feet of fence will she need?

4. What is the diameter of a circle having a circumference of 154 inches?

5. What is the area of a circle with a radius of 5 feet? (Use 3.14 for π.)

6. What is the diameter of the circle in problem number 5?

7. What is the circumference of the circle in problem number 5?

8. Find the area of a circular pool having a diameter of 60 feet. (Use 3.14 for π.)

9. What is the radius of a circle having a circumference of 66 inches? (Hint: Find the diameter first.)

10. Find the circumference of a circle having a diameter of 90 meters. (Use 3.14 for π.)

EXAMPLE 1: Find the circumference of a circle with a diameter of 22.5 inches.

$$C = \pi d$$
$$C = 3.14(22.5)$$
$$C = 70.65 \text{ inches}$$

EXAMPLE 2: Find the diameter of a circle having a circumference of 110 feet.

1. Start with the formula for finding the circumference of a circle. $\quad C = \pi d$

2. Find the formula for d by dividing both sides of the equation by π. $\quad \frac{C}{\pi} = \frac{\pi d}{\pi}$

3. Substitute known values. $\quad \frac{110}{\frac{22}{7}} = d$

$$35 \text{ ft.} = d$$

The formula for the area of a circle is π times the radius squared. That is:

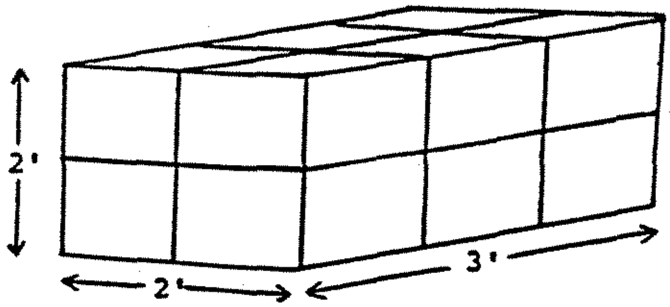
$$A = \pi r^2$$

EXAMPLE 3: Find the area of a circle having a radius of 4 feet.

$$A = \pi r^2$$
$$A = \frac{22}{7}(4)^2$$
$$A = \frac{22}{7}(16)$$
$$A = 50\frac{2}{7}$$

VOLUME

VOLUME is the amount of space taken up by a three-dimensional figure. Volume is measured in cubic inches, cubic feet, cubic centimeters, etc. The volume of a three-dimensional figure can usually be found by multiplying its top surface area by its depth (also known as its height).



The top surface area of the box shown above is found by using the formula: $A = lw$. Using the figures of the box above, this would become $A = 3(2)$, or 6 square feet. The volume is the surface area times the depth, or 6(2), which equals 12 cubic feet.

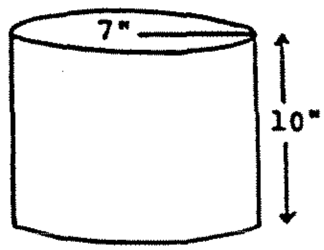
The formula for a rectangular solid can then be given as: $V = lwh$.

The top surface area of a cylinder is found by using the formula: $A = \pi r^2$.

$$A = \pi r^2$$

$$A = \frac{22}{7}(7)^2$$

$$A = 154 \text{ sq. in.}$$



The volume of a cylinder is found by multiplying the top surface area (154 square inches) by the depth (10 inches), which equals 1,540 cubic inches. The formula can be given as:

$$V = \pi r^2 h$$

$$V = \frac{22}{7}(7)^2(10)$$

$$V = (154)(10)$$

$$V = 1{,}540 \text{ cubic inches}$$

EXERCISE 21 (Use 3.14 for π unless otherwise noted.)

1. A swimming pool measures 20' by 40' and is 5' deep. How many cubic feet of water are needed to fill the pool?

2. A circular pool with a radius of 10 feet and a depth of 3.5 feet requires how much water to fill? (Use $\frac{22}{7}$ for π.)

3. A circular hole measures 15 feet across and 14 feet deep. How much dirt is needed to fill the hole?

4. The inside dimensions of a trunk are 36" x 20" x 12". How many cubic inches will the trunk hold?

5. How many cubic inches are in a cubic foot?

6. A drum has a 30" diameter and is 36" tall. What is its volume?

7. A cylinder-shaped oil tank has a radius of 20 meters and is 30 meters high. How many cubic meters of oil will the tank hold?

8. What is the volume of a cube measuring 4" x 4" x 4"?

9. A freight car measures 60' by 20' by 15'. How many cubic feet of cargo can it hold?

10. How many crates measuring 2' by 3' by 1' will fit in the freight car in problem number 9?

GEOMETRY REVIEW

13. 1) A reflex angle has more than _____°.

2) An acute angle has less than _____°.

3) A straight angle has _____°.

4) A right angle has _____°.

5) An obtuse angle has less than 180° but more than _____°.

6) A circle has _____°.

14. 1) Angle a and angle b are complementary. Angle A = 38°. What does angle b equal?

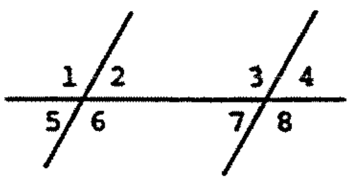
2) Angle b is vertical to angle c. If angle c = 40°, what does angle b equal?

3) Angle 1 and angle 2 are supplementary. Angle 1 = 60°. What does angle 2 equal?

4) Two angles are supplementary. One angle is 8 times the other. What are the two angles?

5) Angle R is complementary to angle K. Angle R is 3 times angle K plus 10°. What are the two angles?

15. Use the figure below to answer questions 1-5. AB//CD.



1) What angle corresponds to angle 2?

2) Which angle is the alternate exterior of angle 4?

3) Which angle is the alternate interior of angle 3?

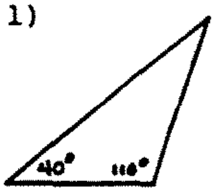
4) If angle 1 = 120°, how many degrees are there in angle 4?

5) If angle 5 is half of angle 6, how many degrees are in angles 5 and 6?

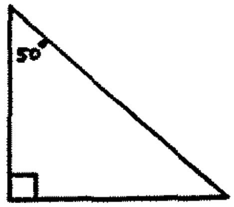
16A. For questions 1-6, write:

a) the value of the unmarked angles
b) the name of the triangle (equilateral, isosceles or scalene)

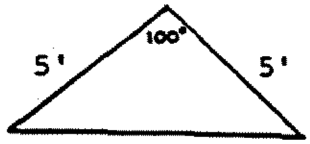
1)



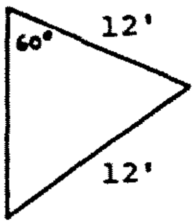
2)



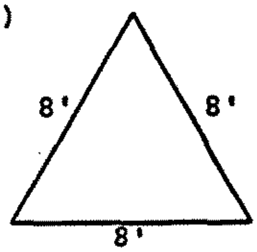
3)



4)

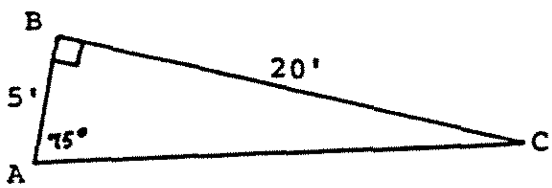


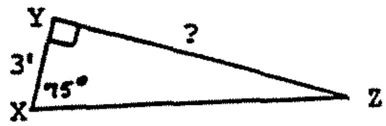
5)



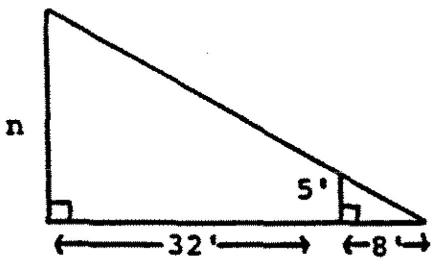
6)

16B. 1) In the figures shown below, how long is side YZ?

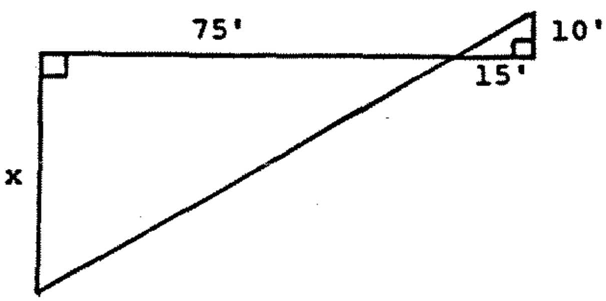




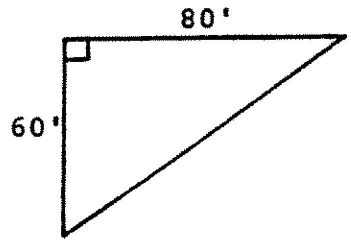
2) Find the length of side n in the figure shown below.



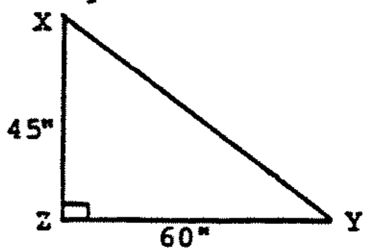
3) Find the length of side x in the figure shown below.



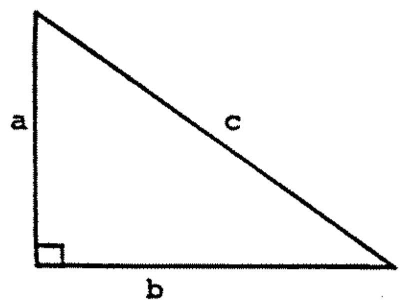
16C. 1) In the triangle given below, what is the length of the hypotenuse?



2) What is the length of XY in the triangle shown below?



3) What is the length of side a in the triangle shown below if c = 40" and b = 32"?



17. 1) In rectangle ABCD, AB//CD and AC//BD. If AB = 95', what is the length of CD?

2) In parallelogram QRST, $\angle$Q = $\angle$S. If $\angle$Q = 50°, how many degrees does $\angle$R equal?

3) In square WXYZ side WX = 15' and $\angle$Y = 90°. What is the length of side XY? How many degrees are there in $\angle$Z?

4) One angle of a rhombus is 50°. What are the other angles?

5) How many parallel pairs does a trapezoid have?

18. 1) What is the perimeter of a rhombus having one side of 15 inches?

2) What is the perimeter of a 12' x 15' rectangle?

3) A parallelogram has parallel sides of 8' and 10', and a height of 5'. What is its perimeter?

4) An equilateral triangle measures 15' on one side. What is its perimeter?

5) The base of an isosceles triangle is 14". One of its two equal sides is 20" and its height is 16" What is its perimeter?

19. 1) Blacktop sealer covers 250 sq. ft. per can. How many cans does Jim need to cover a blacktop measuring 10' x 120'? (Round off to a whole number.)

2) How many square feet of blacktop could Jim cover with what's left over in problem number 1?

3) Theresa has a yard in the shape of a trapezoid. One of the parallel sides measures 50' and the other is 70'. The distance between the two parallel sides is 100 feet. How many square feet of yard does she have?

4) What is the area of a right triangle with legs of 50' and 200'?

5) A parallelogram has a base of 90 feet, sides of 30 feet, and a height of 20 feet. What is its area?

20. Use 3.14 for π.

1) What is the area of a circle having a radius of 1.2 meters?

2) What is the diameter of a circle having a circumference of 60 feet? (Round off to nearest 10th.)

3) What is the circumference of a circular pool having a radius of 20'?

4) The diameter of a half-dollar is 30 mm. What is its circumference?

5) Find the area of a half-dollar.

21. 1) A stack of half-dollars is 50 mm high. What is its volume? (Remember the radius of a half-dollar is 15 mm.)

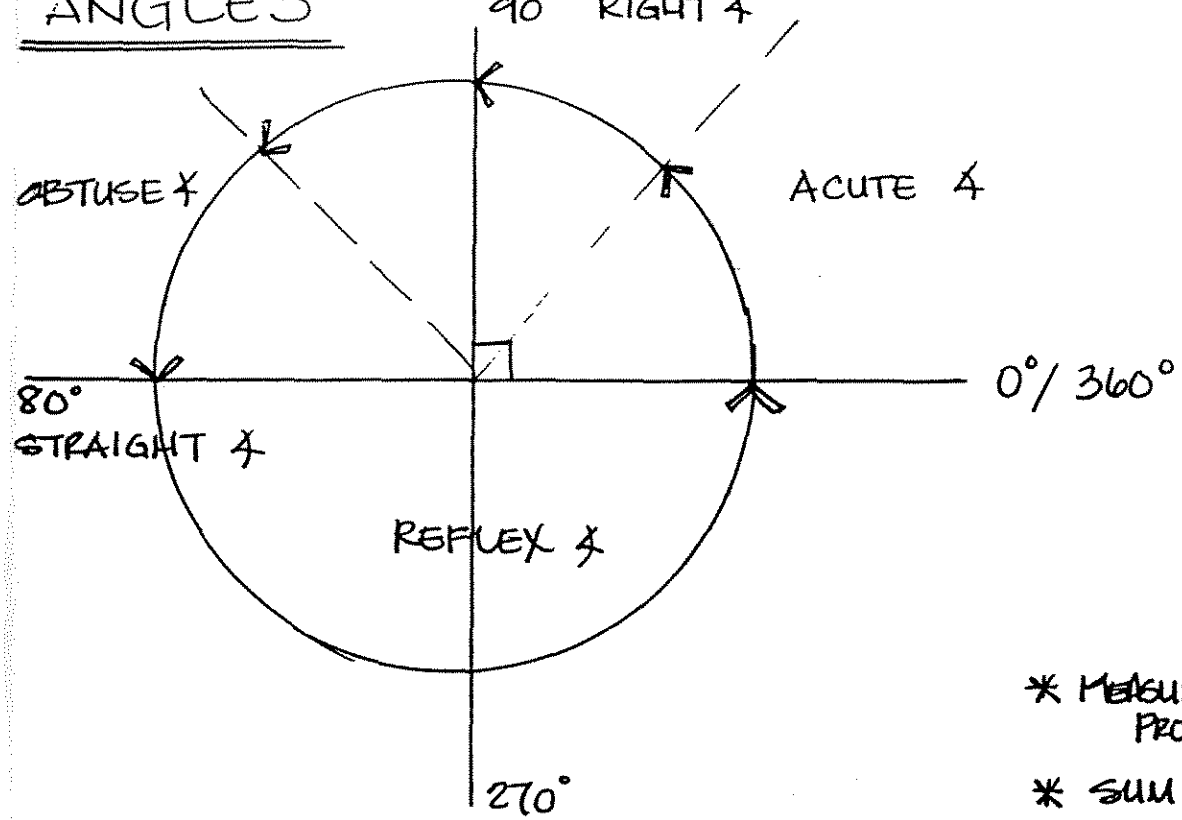
2) A tin box measures 5" by 3 $\frac{1}{2}$" by 2 $\frac{1}{2}$". What is its volume?

3) What is the volume of a smoke stack having a height of 200 feet and a radius of 2 feet?

4) A cup having a 3" diameter and 4" of height will hold how much coffee?

5) The measurements of a filing cabinet are 4' by 1 $\frac{1}{2}$' by 2'. What is its volume?

ANGLES

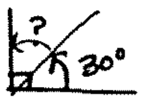


* MEASURE ∡'S WITH A PROTRACTOR

* SUM OF ANGLES = 360°

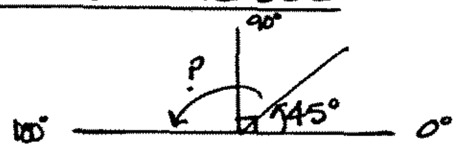
ACUTE < 90°
RIGHT = 90°
OBTUSE > 90° < 180
STRAIGHT = 180°
REFLEX > 180°

COMPLIMENTARY ANGLES — ∡ 1 + ∡ 2 = 90°



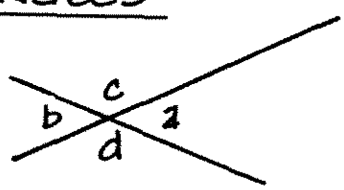
$30° + x = 90°$
$x = 90° - 30°$
$x = 60°$

SUPPLEMENTARY ANGLES — ∡ 3 + ∡ 4 = 180°



$45° + x = 180°$
$x = 180° - 45°$
$x = 145°$

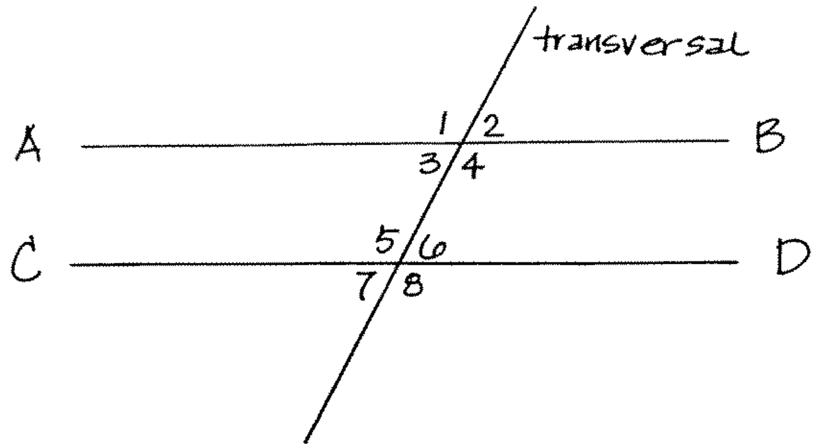
VERTICAL ANGLES



∡ a = ∡ b
∡ c = ∡ d

∡b + ∡c = 180
∡a + ∡d = 180°

CORRESPONDING ANGLES



$AB \parallel CD$

∡'s 1 – 4 ARE VERTICAL ANGLES
∡ 5 – 8

BECAUSE THE LINES ARE PARALLEL THE CORRESPONDING ANGLES ARE EQUAL.

$$\angle 1 = \angle 5$$
$$\angle 2 = \angle 6$$
$$\angle 3 = \angle 7$$
$$\angle 4 = \angle 8$$

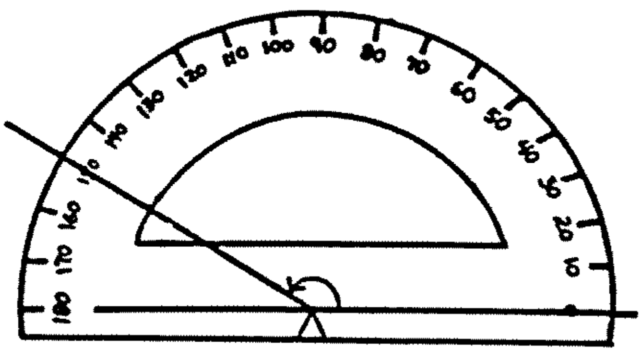
IF ANGLE 2 = 50° WHAT ARE THE OTHER SEVEN ANGLES?

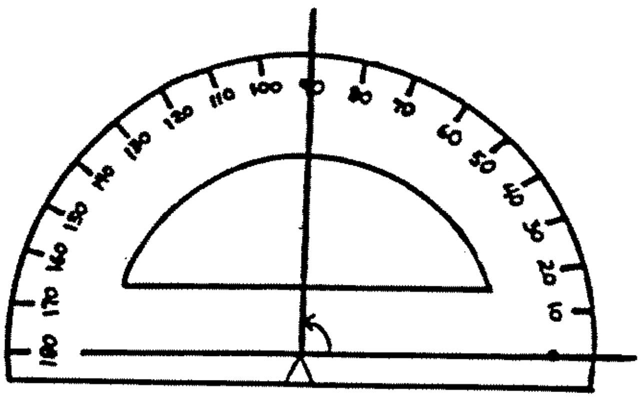
GEOMETRY ANSWERS

EXERCISE 13

Measure each angle. State how many degrees it has and the letter corresponding to its name:

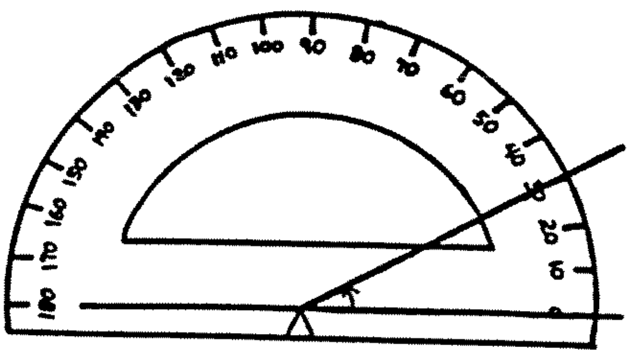
a. acute
b. right
c. obtuse
d. straight
e. reflex

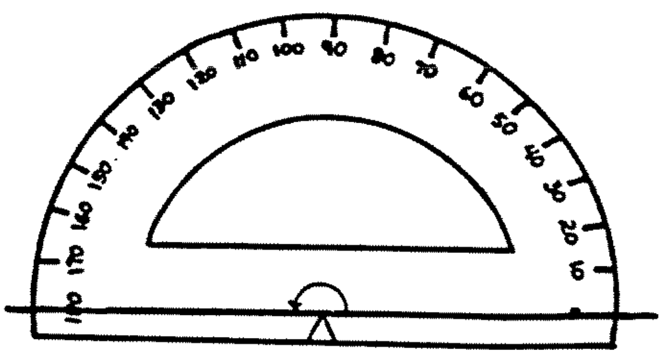




1. 150 degrees; c. angle
obtuse

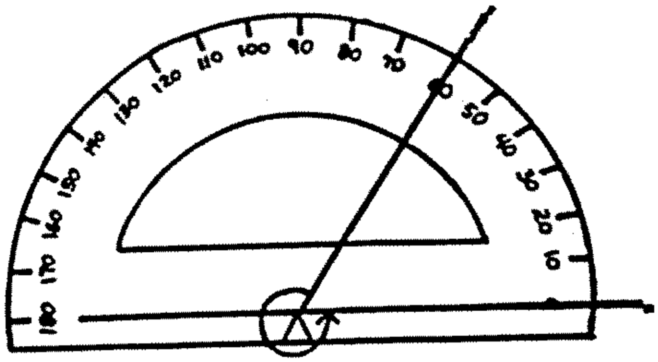
2. 90 degrees; b angle
right





3. 30 degrees; a angle
acute

4. 180 degrees; d angle
straight



5. 300 degrees; e angle
reflex

Match each angle with the letter corresponding to its name.

6. C 125°   a. acute

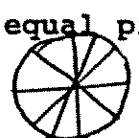
7. e 270°   b. right

8. a 80°   c. obtuse

9. d 180°   d. straight

10. b 90°   e. reflex

11. A round pizza is divided into 10 equal pieces. Each piece is cut at what angle?



360° ÷ 10 = 36°

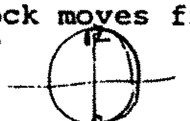
12. How many degrees are in $\frac{1}{8}$ of a circle?

360/8 = 45°

13. A circle is cut in fourths. At what angle is each piece cut?

90°   360/4

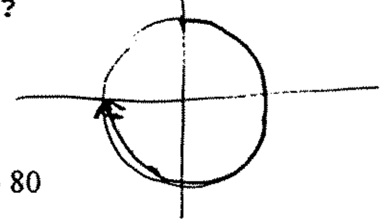
14. When the big hand on a clock moves from 12 to 6, how many degrees does it pass?



180°

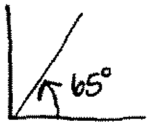
15. When the big hand on a clock moves from 12 to 9, how many degrees does it pass?

270°



EXERCISE 14

1. If $\measuredangle 1$ and $\measuredangle 2$ are complementary, and $\measuredangle 1$ is 65°, what is $\measuredangle 2$?



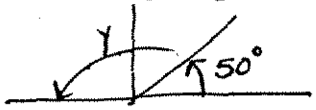
$\measuredangle 1 = 65°$

$65° + \measuredangle 2 = 90°$

$\measuredangle 2 = 90° - 65°$

$\measuredangle 2 = 25°$

2. Angles x and y are supplementary. If angle x = 50° what is angle y?



$x + y = 180°$

$60° + y = 180°$

$y = 180° - 50°$

$y = 130$

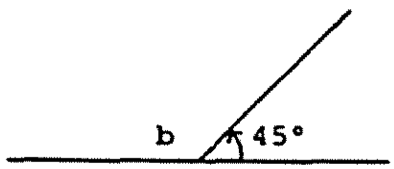
$y = 130$

3. Angle m and angle p are vertical angles. If angle m = 45°, what does angle p equal?

$m = 45°$ –

$p = 45°$

4. In the figure to the right, write an algebraic equation to find $\measuredangle b$, then solve for b.

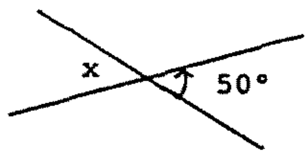


$45° + b = 180°$

$b = 180° - 45°$

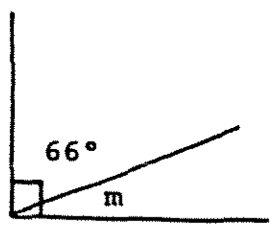
$b = 135°$

5. In the figure to the right, what is $\measuredangle x$?



$x = 50°$

6. In the figure to the right, write an algebraic equation and solution for $\measuredangle m$.

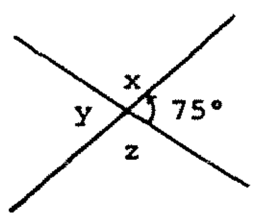


$m + 66° = 90°$

$m = 90° - 66°$

$m = 24°$

7. In the figure to the right, what are the values of angles x, y and z?



$y = 75°$

$x + z = 180 - 75°$

$x = 105$

$y = 105$

8. Two angles are supplementary. One angle is twice the other plus 30°. What are the two angles?

$x + 2x + 30 = 180$
$3x = 180 - 30$
$3x = 150$
$x = 50$

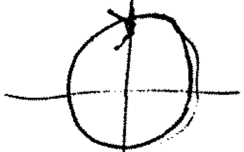
50 and 130

9. Two angles are complementary. If one angle is twice the other, what are the two angles?

$x + 2x = 90$
$3x = 90$
$x = 30$

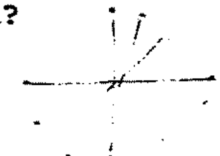
$\cancel{x}$ – 30°
– 60°

10. When the big hand on a clock moves from 12 back to 12 (one complete revolution), how many degrees does it pass through?



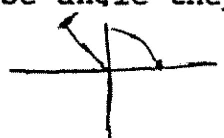
360°

11. When the small hand on a clock moves from 1 to 2, how many degrees does it pass through?



$360/12 = 30°$

12. The big hand on a clock is on 11 and the small hand is on 3. The obtuse angle they form has how many degrees?

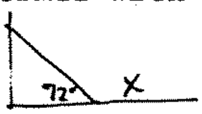


$30° \times 4 = 120°$

13. The angles formed by the Red Cross each have how many degrees?

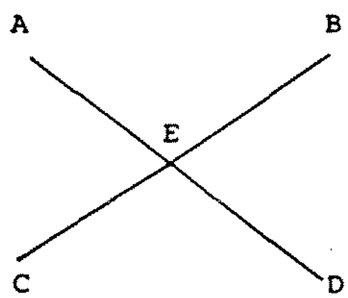
90°

14. A ladder leaning against a house forms a 72° angle with the ground. What is the size of the supplementary angle formed with the ground?



$x + 72° = 180$
$x = 180 - 72$
$x = 108$

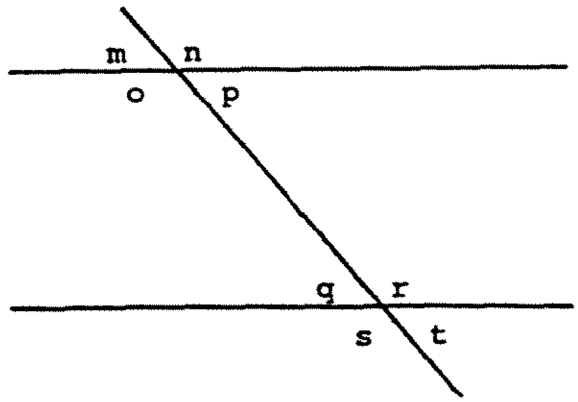
15. In the figure to the right, what angle is vertical to ∡AEC?



∡BED

EXERCISE 15

Use the figure below to answer questions 1-5.



1. What angle corresponds to $\measuredangle$ o? angle s

2. What angle is an alternate exterior to $\measuredangle$ n? angle s

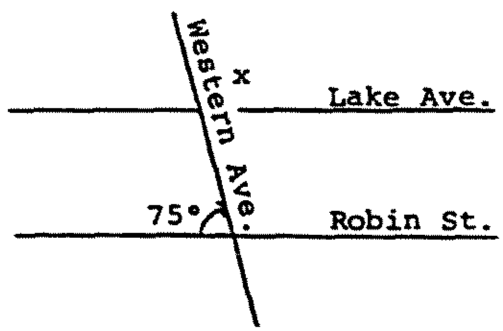
3. What angle is the alternate interior to $\measuredangle$ p? angle q

4. If $\measuredangle$ m = 50°, what does $\measuredangle$ s equal? 130°

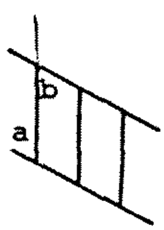
5. If $\measuredangle$ o = 130°, what does $\measuredangle$ t equal? 50°

6. Western Avenue crosses Lake Avenue and Robin Street at an angle of 75° as shown below. What is the value of $\measuredangle$ x?

105°

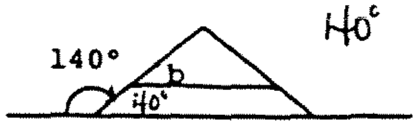


7. If $\measuredangle$ a = 65° in the hand railing shown to the right, what is $\measuredangle$ b? 65°

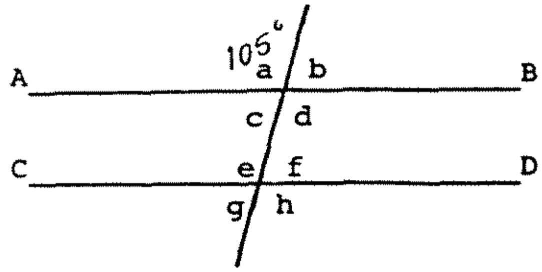


8. In the letter A pictured to the right, what is the value of $\measuredangle$ b?

40°



Use the figure below to answer questions 9-15. AB//CD; a = 105°.



9. What is $\measuredangle$b? 75°

10. What is $\measuredangle$c? 75°

11. What is $\measuredangle$d? 105°

12. What is $\measuredangle$e? 105°

13. What is $\measuredangle$f? 75°

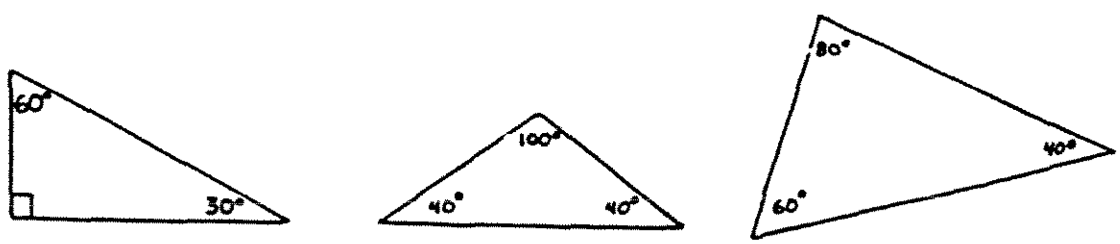
14. What is $\measuredangle$g? 75°

15. What is $\measuredangle$h? 105°

TRIANGLES

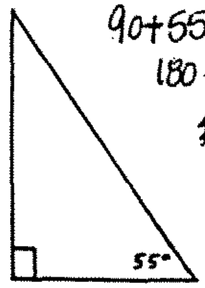
A TRIANGLE is a plane (flat) figure with 3 sides and 3 angles. The symbol for triangle is "Δ". The 3 angles of a triangle always add up to 180° no matter what the shape of the triangle.

EXAMPLES:

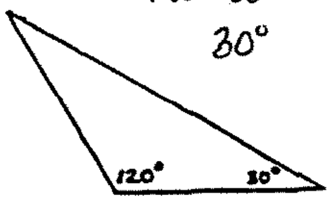


EXERCISE 16A

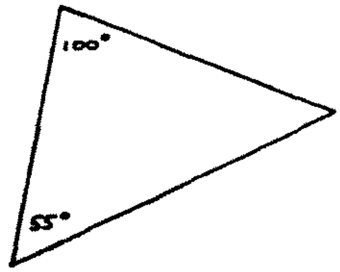
Find the value of each unmeasured angle in the triangles below.

1. 

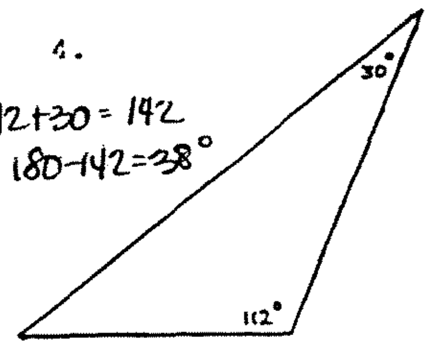
90+55= 145
180-145=
~~135°~~
35°

2. 

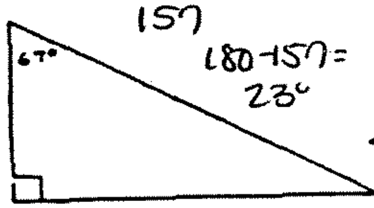
180-150=
30°

3. 

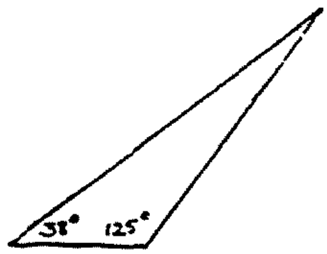
180 − 155 = 25°

4. 

112+30 = 142
180-142=38°

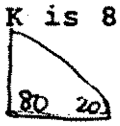
5. 

67+90 =
157
180-157=
23°

6. 

125+38 =
163
180-163=
17°

7. In triangle RJK, ∡K is 80° and ∡J is 20°. What is the value of ∡R?

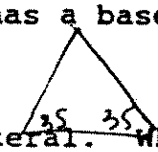


180-100 = 80°

8. In triangle ABC, ∡A is 40° and ∡B is a right angle. What is the value of ∡C?

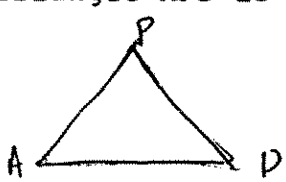
90+40 = 130   180-130 = 50°

9. An isosceles triangle has a base angle of 35°. What is the vertex angle?



35+35 = 70   180-70= 110

10. Triangle APD is equilateral. What is the value of ∡A?

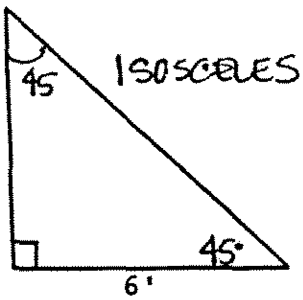


60°

For questions 11-16, write:

(1) the value of the unnamed angle(s)
(2) the name of the triangle (equilateral, isosceles, or scalene)

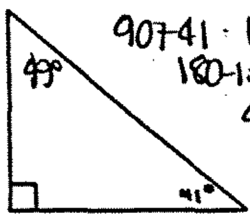
11.



ISOSCELES

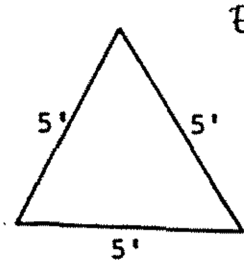
12.

SCALENE
RIGHT



90+41 = 131
180-131 =
49

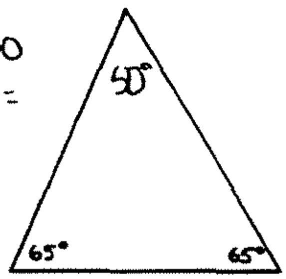
13.



EQUILATERAL
60°

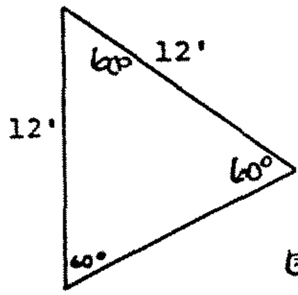
14.

$65 \times 2 = 130$
180-130 =
50



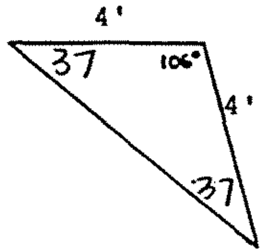
ISOCELES

15.



EQUILATERAL

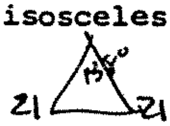
16.



180 -
106
74/2
37

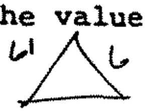
ISOCELES

17. The base angle of an isosceles triangle is 21°. What is the vertex angle?



$21 \times 2 = 42$
180 - 42 = 138

18. The side of an isosceles triangle opposite a base angle is 6'. What is the value of the side opposite the other base angle?

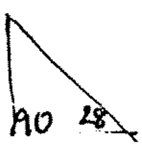


6'

19. In triangle ABC, ∡ A = 60° and ∡ B = 60°. Side AB = 40 feet. What does side BC measure?

40'

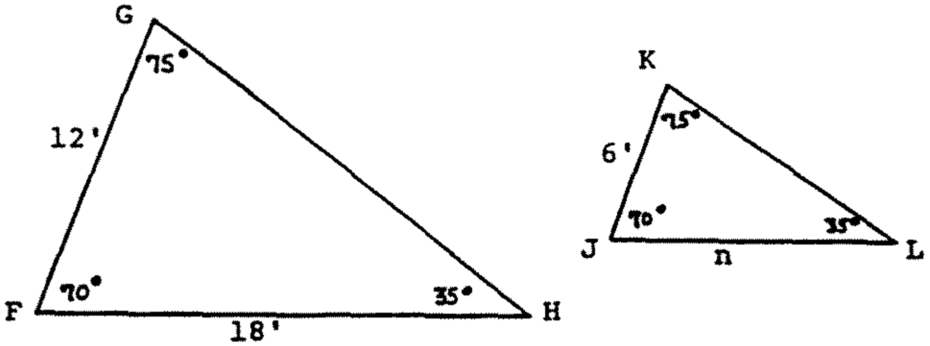
20. In a scalene right triangle, one angle is 28°. What is the value of the other acute angle?



90+28 = 118 180-118 = 62°

EXERCISE 16B

1. $\triangle$ FGH $\sim$ $\triangle$ JKL. What is the length of side n?



$\frac{GF}{KJ} = \frac{FH}{JL}$

$\frac{12}{6} = \frac{18}{n}$

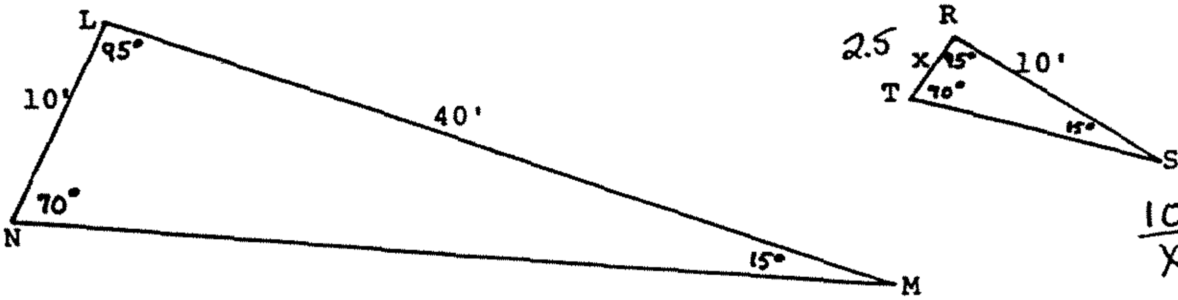
$12n = 18 \times 6$

$12n = 108$

$n = 108/12$

$n = 9$

2. $\triangle$ LMN $\sim$ $\triangle$ RST. What is the length of side x?



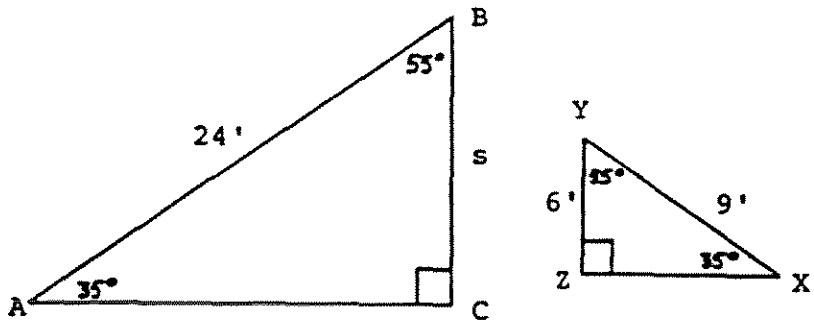
2.5

$\frac{LN}{RT} = \frac{LM}{RS}$

$\frac{10}{x} = \frac{40}{10}$ $\quad 40x = 100$

$x = 2.5$

3. $\triangle$ ABC $\sim$ $\triangle$ XYZ. What is the length of side s?



$\frac{AB}{XY} = \frac{BC}{YZ}$

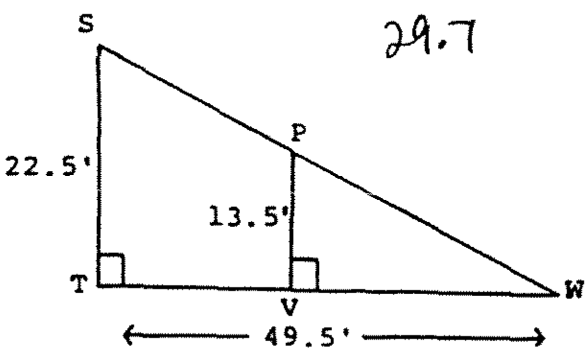
$\frac{24}{9} = \frac{s}{6}$ $\quad 9s = 144$

$s = 144/9$

$s = 16$

4. ST = 22.5, PV = 13.5, and TW = 49.5. What is the length of VW?

29.7



$\frac{ST}{PV} = \frac{TW}{VW}$

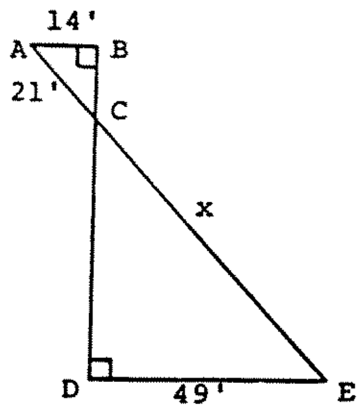
$\frac{22.5}{13.5} = \frac{49.5}{x}$

$22.5(x) = 49.5 \times 13.5$

$22.5(x) = 668.25$

$x = 29.7$

5. AB = 14, AC = 21, and DE = 49. What is the value of CE?



$$\frac{AB}{DE} = \frac{AC}{CE}$$

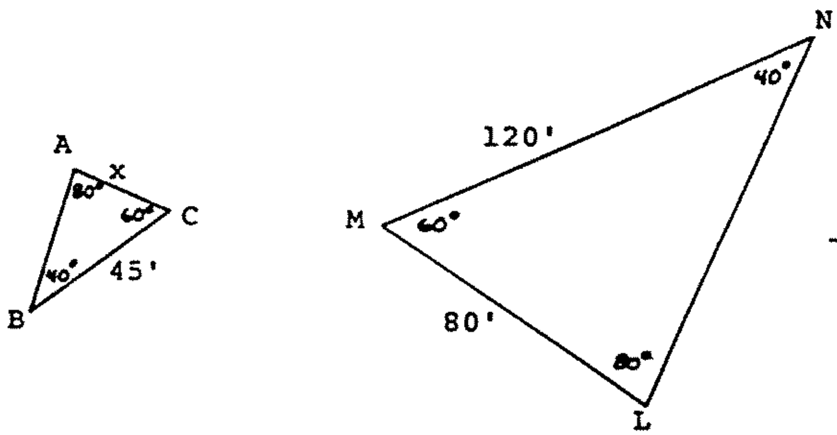
$$\frac{14}{49} = \frac{21}{x}$$

$$14x = 49(21)$$

$$14x = 1029$$

$$x = 73.5$$

6. Given the figures shown below, find side x.



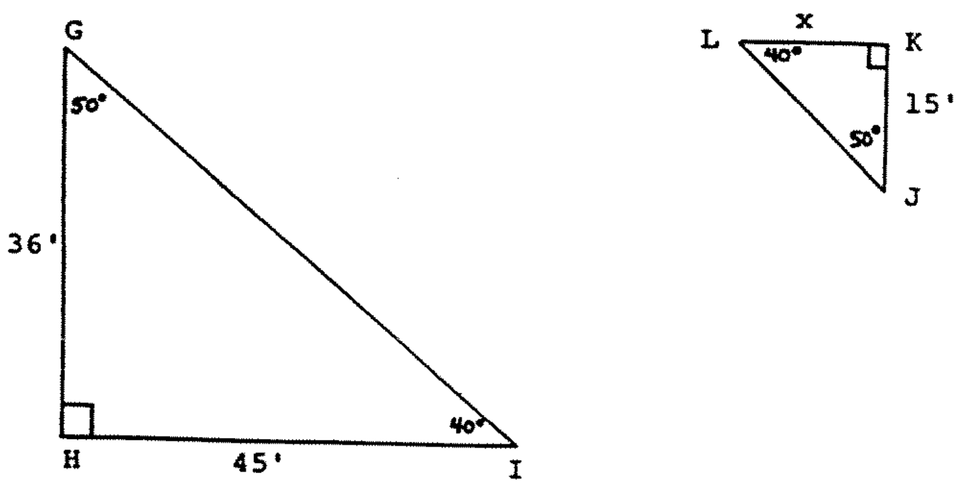
$$\frac{AC}{ML} = \frac{BC}{MN}$$

$$\frac{x}{80} = \frac{45}{120}$$

$$120x = 3600$$

$$x = 30$$

7. Given the figures shown below, find side x.



$$\frac{LK}{HI} = \frac{KJ}{GH}$$

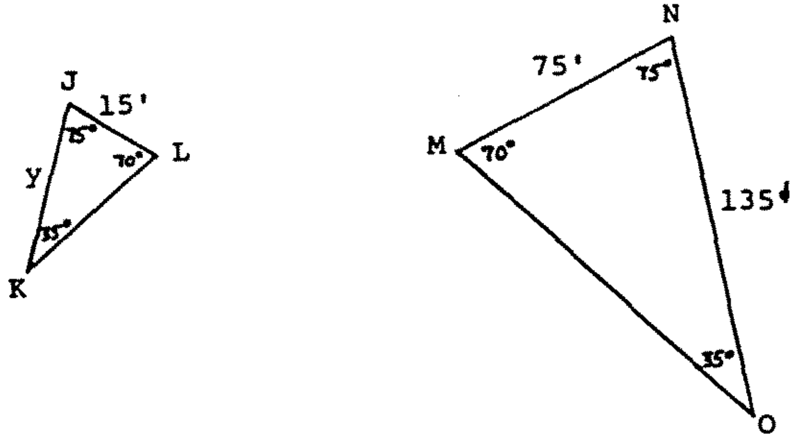
$$\frac{x}{45} = \frac{15}{36}$$

$$36x = 675$$

$$x = 18.75$$

8. Find the value of side y in the figures shown below.

NOTE: $\measuredangle J = 75°$, $\measuredangle K = 35°$, and $\measuredangle L = 70°$.



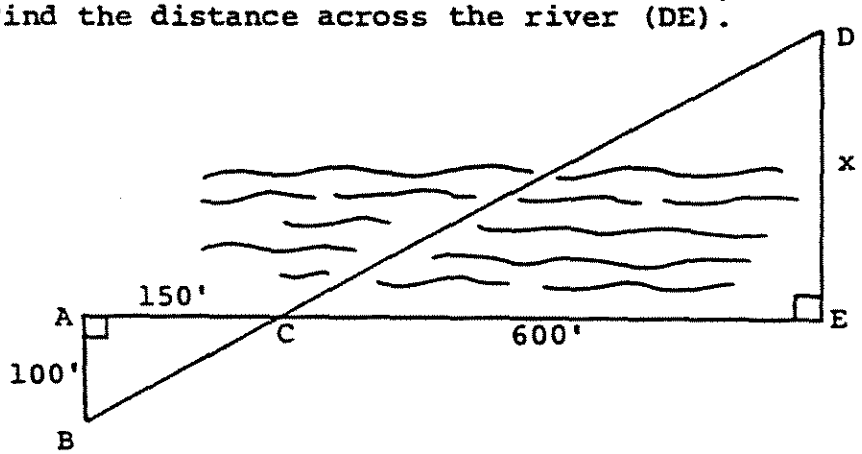
$$\frac{JK}{NO} = \frac{JL}{MN}$$

$$\frac{y}{135} = \frac{15}{75}$$

$$75y = 2025$$

$$y = 27$$

9. To find the distance across the Hudson River, Tom measured the distances shown in the diagram below. Find the distance across the river (DE).



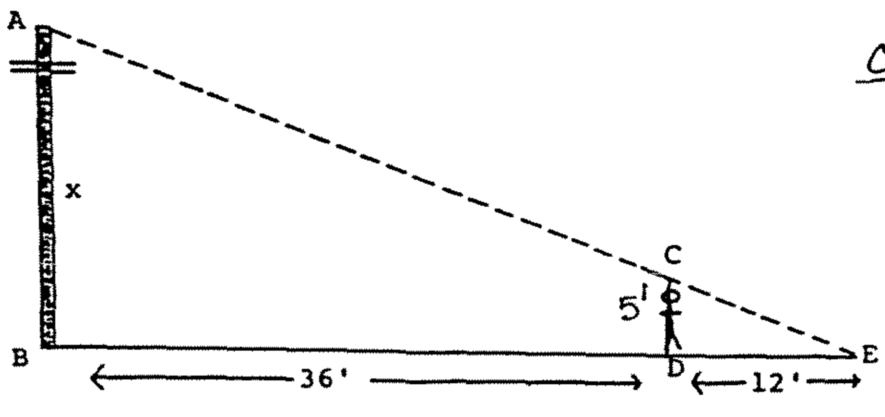
$$\frac{AB}{DE} = \frac{AC}{CE}$$

$$\frac{100}{x} = \frac{150}{600}$$

$$150x = 6000$$

$$x = 40$$

10. To measure the height of a phone pole, Maria, who is 5 feet tall, measured her shadow and the shadow cast by the pole as shown in the diagram below. How tall is the phone pole?



$$\frac{CD}{AB} = \frac{DE}{BE}$$

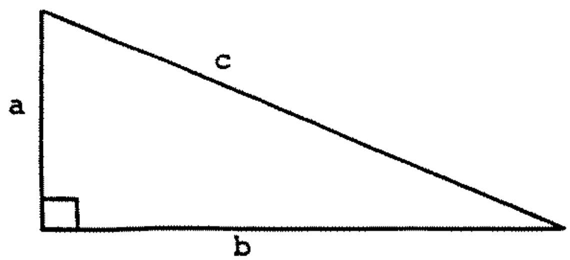
$$\frac{5}{x} = \frac{12}{48}$$

$$12x = 240$$

$$x = 20$$

EXERCISE 16C

1. In the triangle shown below, find side c, if side a = 5' and side b = 12'.



$c^2 = a^2 + b^2$

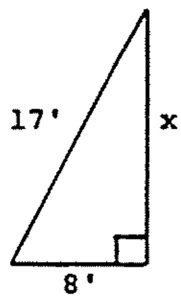
$c^2 = 5^2 + 12^2$

$c^2 = 25 + 144$

$c^2 = 169$

$c = 13$

2. In the triangle shown below, find the length of the unknown leg (x).



$c^2 = a^2 + b^2$

$17^2 = 8^2 + b^2$

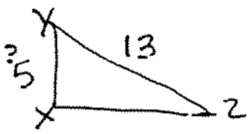
$289 = 64 + b^2$

$b^2 = 289 - 64$

$b^2 = 225$

$b = 15$

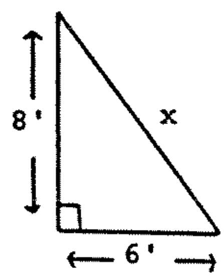
3. In triangle XYZ, the hypotenuse (YZ) is 13 feet long. Side XY is 5 feet long. What is the length of side XZ?



$13^2 = 5^2 + b^2$ $169 = 25 + b^2$ $b^2 = 169 - 25$

$b^2 = 144$ $b = 12$

4. In the right angle shown below, find the length of the hypotenuse.



$c^2 = a^2 + b^2$

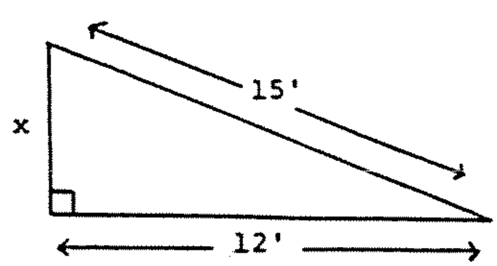
$c^2 = 8^2 + 6^2$

$c^2 = 64 + 36$

$c^2 = 100$

$c = 10$

5. Find the length of the missing leg in the triangle shown below.



$15^2 = 12^2 + x^2$

$225 = 144 + x^2$

$x^2 = 225 - 144$

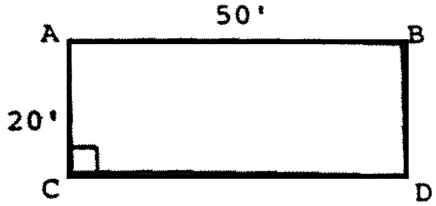
$x = 81$

$x = 9$

EXERCISE 17

1. In rectangle ABCD, side AB = 50' and side AC = 20'. What is the length of side BD?

AC = BD   20'



2. In rectangle ABCD, what is the length of side CD?

AB = CD   50'

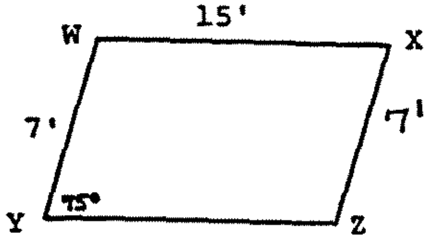
3. In rectangle ABCD, how many degrees are in ∡A?

ALL ANGLES ARE EQUAL   90°

4. In parallelogram WXYZ, side WX = 15', side WY = 7', and ∡Y = 75°. What is the length of side XZ?   WY = XZ

$75° \times 2 = 150°$ 7'

$360° - 150° = 210°$



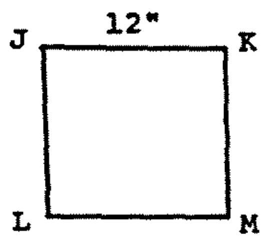
5. In parallelogram WXYZ, how many degrees are in ∡Z?

$210° \div 2 = 105°$

6. In square JKLM, side JK is 12". What is the length of side KM?

ALL SIDES ARE EQUAL IN A SQUARE.

KM = 12"



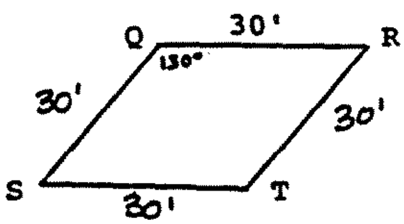
7. In square JKLM, how many degrees are there in ∡L?

ALL ANGLES ARE EQUAL - 90°

8. In rhombus QRST, side QR = 30' and ∡Q = 130°. What is the length of side QS?

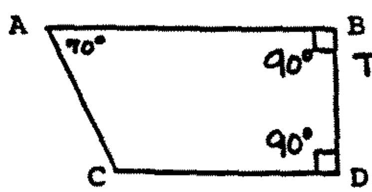
ALL SIDES ARE EQUAL.

QS = 30'



9. In rhombus QRST, how many degrees are in ∡T?   130°

10. In the figure to the right, ∡B and ∡D are right angles, ∡A = 70°, and AB//CD. What is the figure called?



TRAPEZOID

11. What are AB and CD called in problem 10?

BASES

12. How many degrees are there in ∡C in problem 10?

$90° + 90° + 70° + x = 360°$

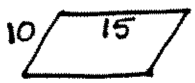
$250° + x = 360°$

$x = 360° - 250°$

$x = 110°$

EXERCISE 18

1. One side of a parallelogram is 15 feet and the other is 10 feet. What is its perimeter?



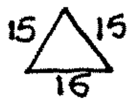
$15(2) + 10(2) = P$ $\quad$ $P = 50$

$30 + 20 = P$

2. One side of a rhombus is 25". What is its perimeter?

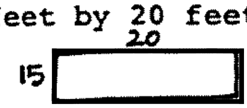
$25'' \times 4 = 100''$

3. One side of an equilateral triangle is 15 feet. What is its perimeter?



$15 \times 3 = 45$

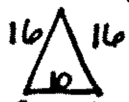
4. What is the perimeter of a rectangle that measures 15 feet by 20 feet?



$20(2) + 15(2) = P$ $\quad$ $P = 70$

$40 + 30 = P$

5. One of the two equal sides of an isosceles triangle measures 16 meters and the third side measures 10 meters. What is the triangle's perimeter?



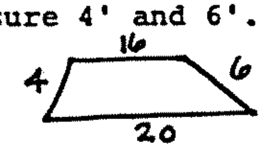
$16 + 16 + 10 = P$

$42 = P$

6. The bases of a trapezoid measure 20' and 16' respectively. The other two sides measure 4' and 6'. What is the perimeter of the trapezoid?

$16 + 20 + 4 + 6 = P$

$P = 46$

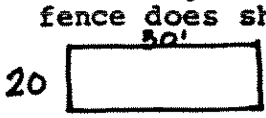


7. The legs of a right triangle measure 6' and 8'. What is the perimeter of the triangle? (Hint: First find the length of the hypotenuse.)

$c^2 = a^2 + b^2$ $\quad$ $c^2 = 100$ $\quad$ $10 + 8 + 6 = P$

$c^2 = 8^2 + 6^2$ $\quad$ $c = 10$ $\quad$ $P = 24$

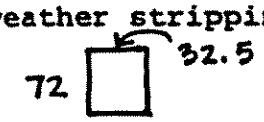
8. Maria's garden measures 20' by 30'. How many feet of fence does she need to enclose the garden? Perimeter



$20(2) + 30(2) = P$ $\quad$ $P = 100$

$40 + 60 = 100$

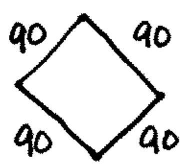
9. A window measures 72" by 32.5". How many inches of weather stripping are needed to insulate the window?



$72(2) + 32.5(2)$

$144 + 65 = 209'' = P$

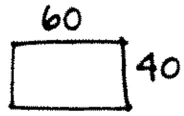
10. A baseball diamond is actually a square measuring 90' on a side. How many feet must a player run in order to score? (To score a player must run around the "diamond".)



$P = 90 \times 4 = 360$

EXERCISE 19

1. What is the area of a yard measuring 60' by 40'?



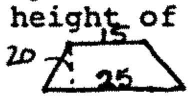
Area = b×h   60×40 = 2400 sq. ft.

2. The area of a triangle is 1,280 sq. ft. If the base is 16 feet, what is the height?

Area = 1/2 b×h   1/2 (16) × h = 1280
8 × h = 1280   h = 1280/8
h = 160

3. Find the area of a trapezoid that has bases of 15 meters and 25 meters, and a height of 20 meters.

Area = 1/2 (b_1 + b_2) × h



1/2 (15+25) × 20 = A   A = 20×20
1/2 (40) × 20 = A   A = 400

4. Felipa wants to buy a carpet for her living room which measures 12' by 21'. How many square yards of carpeting will she need? (Hint: Change feet to yards before multiplying).

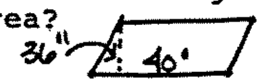
Area = b×h

21×12 = 252 sq ft.   252/9 = 28 sq. yds.

5. What is the area of an isosceles triangle that has a base of 20 feet, height of 16 feet, and equal sides of 18 feet?

1/2 b×h = Area   1/2 (20) × 16
10 × 16 = 160 sq ft.

6. A parallelogram has a height of 36" and a base of 40". What is its area?



Area = b×h
A = 36" × 40"   Area = 1440 sq. in

7. Building costs at the Cape are about $80/sq. ft. of living space. The downstairs living space of a home that Annie and John want to build measures 30' by 40' and the upstairs living space is 30' by 20'. What will it cost to construct the house?

Area = b×h   1st floor 30×40 = 1200
2nd floor 30×20 = 600
1200+600 = 1800 sq ft.
1800 × $80 = $144,000

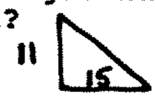
8. How many square feet are in a square yard?

3×3 = 9 sq. ft.

9. How many square inches are in a square foot?

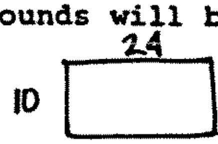
12×12 = 144"

10. A right triangle has legs measuring 15" and 11". What is its area?



1/2 b×h   1/2 (11) × 15
5.5 × 15 = 82.5 sq. in.

11. A cabbage patch measures 10 yards by 24 yards. If a pound of fertilizer will cover 12 square yards, how many pounds will be needed for the cabbage patch?

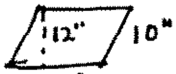


24×10 = 240 sq. yds.
240/12 = 20 lbs.

12. Fritz plans to paper 3 walls each measuring 20' by 10'. If one roll of paper covers 30 sq. ft., how many rolls will he need?

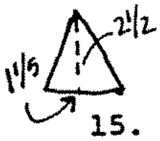
$3(20 \times 10) = 3(200) = 600$
$600 \div 30 = 20$ rolls

13. What is the area of a rhombus having a height of 12" and a side of 10"?



$A = b \times h$ Area $= 10 \times 12 = 120$ sq. in.

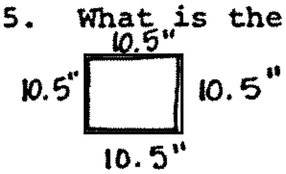
14. A triangle measures $2\frac{1}{2}$ feet in height and has a base of $1\frac{1}{5}$ feet. What is its area?



$.6 \times 2.5 = 1.5$ sq. ft.

Area $= 1/2\, b \times h$ $1/2(1.2) \times 2.5 =$ Area

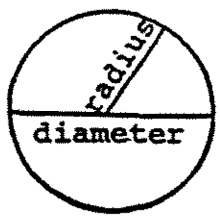
15. What is the area of a square measuring 10.5" on a side?



$10.5" \times 10.5" =$ Area
Area $= 110.25$ sq. in.

CIRCLES

A CIRCLE is a curved, closed, plane (flat) geometric figure. Each point on the curve is an equal distance from the center. The distance from the center to any point on the curve is called the RADIUS. The DIAMETER of a circle is a line passing through the circle's center as shown below.



The distance around a circle is its CIRCUMFERENCE. The circumference of a circle could be thought of as the circle's perimeter.

To find the circumference of a circle, a special number represented by the symbol "π" (pi) is used. π is a Greek letter having a value of 3.14 or $3\frac{1}{7}$ which is usually expressed as the improper fraction $\frac{22}{7}$. (Depending on which one you use, you may get slightly different answers. In the exercises that follow, we indicate which number you should use to get exactly the same answers as we have.) The formula for the circumference of a circle is π times the diameter. That is:

$$C = \pi d$$

(Use 3.14)

EXERCISE 20 (Use $\frac{22}{7}$ for π unless otherwise noted.)

1. Find the circumference of a circle having a diameter of 14 meters.

$C = \pi d$
$C = 3.14(14)$ $C = 43.96$

2. What is the area of a circle with a diameter of 40 inches? Use 3.14 for π. (Note: The radius is $\frac{1}{2}$ the diameter.)

$A = \pi r^2$ $A = 3.14 \times 20''^2$
$A = 3.14 \times 400$ $A = 1256$

3. Cynthia needs to fence in a corral having a diameter of 50 feet. How many feet of fence will she need?

$C = \pi d$ $3.14 \times 50' = C$
$C = 157'$

4. What is the diameter of a circle having a circumference of 154 inches?

$C = \pi d$ $x = 154''/3.14$
$154'' = 3.14(x)$ $x = 49$

5. What is the area of a circle with a radius of 5 feet? (Use 3.14 for π.)

$Area = \pi r^2$ $A = 3.14 \times 5^2$ $A = 78.5$
3.14×25

6. What is the diameter of the circle in problem number 5?

$10'$ $r \times 2 = d$

7. What is the circumference of the circle in problem number 5?

$C = \pi d$ $3.14 \times 10 = C$
$C = 31.40$

8. Find the area of a circular pool having a diameter of 60 feet. (Use 3.14 for π.)

$A = \pi r^2$ $A = 3.14(30)^2$ $900 \times (3.14)$
$A = 2826$

9. What is the radius of a circle having a circumference of 66 inches? (Hint: Find the diameter first.)

$C = \pi d$ $66 = \pi d$ $d = 66/3.14$
$66 = 3.14(d)$ $d = 21$ $21/2 = 10.5 = r$

10. Find the circumference of a circle having a diameter of 90 meters. (Use 3.14 for π.)

$C = \pi d$
$C = 3.14(90)$
$C = 282.6$

EXERCISE 21 (Use 3.14 for π unless otherwise noted.)

1. A swimming pool measures 20' by 40' and is 5' deep. How many cubic feet of water are needed to fill the pool?

5' deep $b \times h \times w$

$20 \times 40 \times 5 = 4000$ cubic ft.

2. A circular pool with a radius of 10 feet and a depth of 3.5 feet requires how much water to fill? (Use $\cancel{22/7}$ 3.14) for π.)

$V = \pi r^2 \times h$

$V = 3.14(10)^2 \times 3.5$

$V = 1099$ cubic ft.

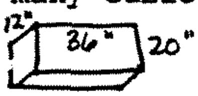
3. A circular hole measures 15 feet across and 14 feet deep. How much dirt is needed to fill the hole?

$V = \pi r^2 \times h$.

$V = 3.14(7.5)^2 \times 14$

$V = 2472.75$ cubic feet

4. The inside dimensions of a trunk are 36" x 20" x 12". How many cubic inches will the trunk hold?



$V = b \times w \times h$

$V = 36 \times 20 \times 12 =$

$V = 8640$ cubic in.

5. How many cubic inches are in a cubic foot?

12" × 12" × 12" = 1728 cubic inches

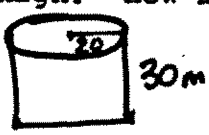
6. A drum has a 30" diameter and is 36" tall. What is its volume?

$V = \pi r^2 \times h$

$V = 3.14(15^2) \times 36"$

$V = 25.434 \text{ in}^3$

7. A cylinder-shaped oil tank has a radius of 20 meters and is 30 meters high. How many cubic meters of oil will the tank hold?



$V = \pi r^2 \times h$

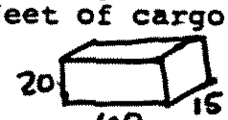
$V = 3.14(20)^2 \times 30$

$V = 37680$ cubic m.

8. What is the volume of a cube measuring 4" x 4" x 4"?

$V = b \times w \times h$ $4 \times 4 \times 4 = 64$ cub. in.

9. A freight car measures 60' by 20' by 15'. How many cubic feet of cargo can it hold?



$V = b \times w \times h$

$V = 20 \times 60 \times 15$

$V = 18,000$ cubic ft.

10. How many crates measuring 2' by 3' by 1' will fit in the freight car in problem number 9?

$2 \times 3 \times 1 = 6$ cubic ft.

18000 / 6 = 3000 crates

GEOMETRY REVIEW

13. 1) A reflex angle has more than 180 °.

2) An acute angle has less than 90 °.

3) A straight angle has 180 °.

4) A right angle has 90 °.

5) An obtuse angle has less than 180° but more than 90 °.

6) A circle has 360 °.

14. 1) Angle a and angle b are complementary. Angle A = 38°. What does angle b equal? $90° - 38° = 52°$

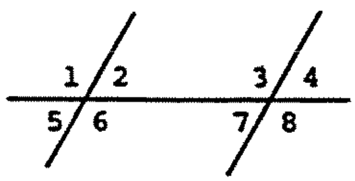
2) Angle b is vertical to angle c. If angle c = 40°, what does angle b equal? b c=40 b=40°

3) Angle 1 and angle 2 are supplementary. Angle 1 = 60°. What does angle 2 equal? 60 2 180-60=120

4) Two angles are supplementary. One angle is 8 times the other. What are the two angles? $x + 8x = 180$ $9x = 180$ $x = 20$ $y = 160$

5) Angle R is complementary to angle K. Angle R is 3 times angle K plus 10°. What are the two angles? 20/70
$x + 3x + 10 = 90$ $4x = 80$ $x = 20$

15. Use the figure below to answer questions 1-5. AB//CD.



1) What angle corresponds to angle 2? Angle 5

2) Which angle is the alternate exterior of angle 4? Angle 5

3) Which angle is the alternate interior of angle 3? Angle 6

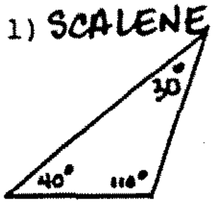
4) If angle 1 = 120°, how many degrees are there in angle 4? 60°

5) If angle 5 is half of angle 6, how many degrees are in angles 5 and 6? 60°

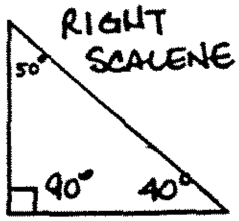
16A. For questions 1-6, write:

a) the value of the unmarked angles
b) the name of the triangle (equilateral, isosceles or scalene)

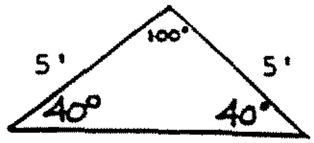
1) SCALENE



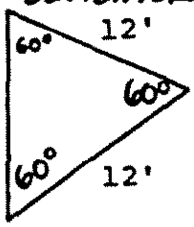
2) RIGHT SCALENE



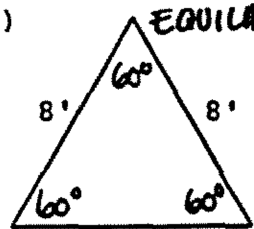
3) ISOSCELES



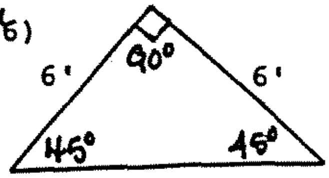
4) EQUILATERAL



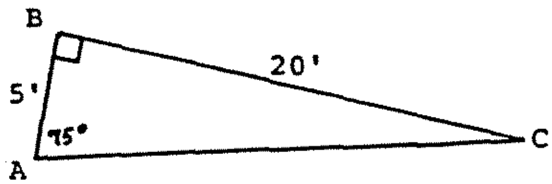
5) EQUILATERAL

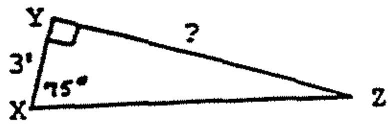


6) RIGHT



16B. 1) In the figures shown below, how long is side YZ?





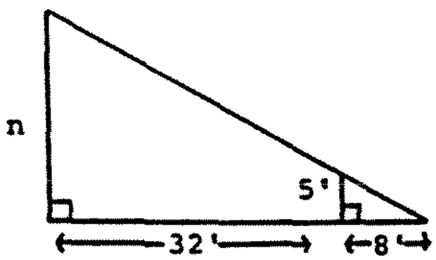
$\frac{BA}{YX} = \frac{BC}{YZ}$

$\frac{5}{3} = \frac{20}{x}$

$5x = 60$

$x = 12$

2) Find the length of side n in the figure shown below.

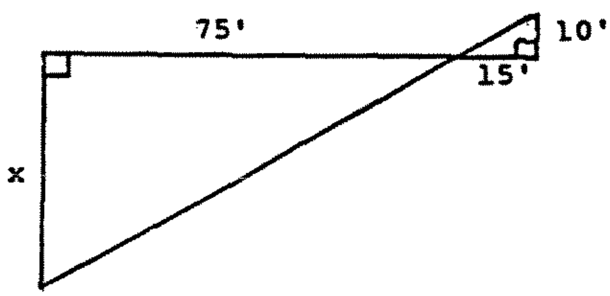


$\frac{5}{n} = \frac{8}{40}$

$200 = 8n$

$n = 25$

3) Find the length of side x in the figure shown below.

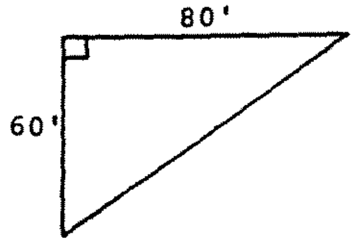


$\frac{10}{x} = \frac{15}{75}$

$750 = 15x$

$x = 50$

16C. 1) In the triangle given below, what is the length of the hypotenuse?



$C^2 = a^2 + b^2$

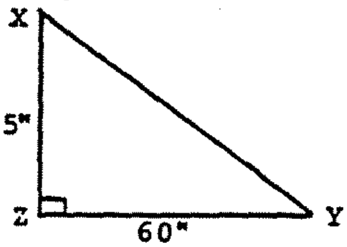
$C^2 = 60^2 + 80^2$

$C^2 = 3600 + 6400$

$C^2 = 10,000$

$C = 100$

2) What is the length of XY in the triangle shown below?



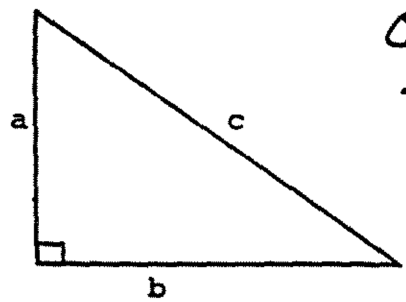
$C^2 = 45^2 \times 60^2$

$C^2 = 2025 + 3600$

$C^2 = 5625$

$C = 75$

3) What is the length of side a in the triangle shown below if c = 40" and b = 32"?



$C^2 = a^2 + b^2$

$40^2 = a^2 + 32^2$

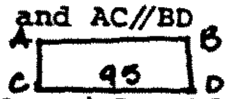
$1600 = a^2 + 1024$

$a^2 = 1600 - 1024$

$a^2 = 576$

$a = 24$

17. 1) In rectangle ABCD, AB//CD and AC//BD. If AB = 95', what is the length of CD?



95' AB=CD

2) In parallelogram QRST, ∡Q = ∡S. If ∡Q = 50°, how many degrees does ∡R equal?

130°

3) In square WXYZ side WX = 15' and ∡Y = 90°. What is the length of side XY? How many degrees are there in ∡Z?

ALL SIDES = 15'

ALL ANGLES = 90°

4) One angle of a rhombus is 50°. What are the other angles?

50×2 = 100 360-100 = 260/2 = 130

5) How many parallel pairs does a trapezoid have?

1 SET.

18. 1) What is the perimeter of a rhombus having one side of 15 inches? $15'' \times 4 = 60''$

2) What is the perimeter of a 12' x 15' rectangle?
$P = 2(L) + 2(W)$ $\quad P = 12(2) + 15(2) =$ $\quad P = 54''$

3) A parallelogram has parallel sides of 8' and 10', and a height of 5'. What is its perimeter? 10 8 h=5
$P = 10(2) + 8(2)$ $\quad P = 36'$

4) An equilateral triangle measures 15' on one side. What is its perimeter? 15 15 15 $\quad 15 \times 3 = 45'$

5) The base of an isosceles triangle is 14". One of its two equal sides is 20" and its height is 16". What is its perimeter? 20 20 14
$P = 20 + 20 + 14$
$P = 54$

19. 1) Blacktop sealer covers 250 sq. ft. per can. How many cans does Jim need to cover a blacktop measuring 10' x 120'? (Round off to a whole number.) $10 \times 120 = 1200$ sq ft. $\quad 1200/250 = 4.8 - 5$ cans

2) How many square feet of blacktop could Jim cover with what's left over in problem number 1? 50 sq. ft.

3) Theresa has a yard in the shape of a trapezoid. One of the parallel sides measures 50' and the other is 70'. The distance between the two parallel sides is 100 feet. How many square feet of yard does she have? $A = 1/2(b_1 + b_2) \times h$ $\quad A = 60 \times 100 = 6000$ 50 100' 70

4) What is the area of a right triangle with legs of 50' and 200'? 200 50 $\quad 1/2\, b \times h = 1/2(200) \times 50$ $\quad 100 \times 50 = 5000$

5) A parallelogram has a base of 90 feet, sides of 30 feet, and a height of 20 feet. What is its area?
20 90 30 $\quad A = b \times h$ $\quad A = 90 \times 20 = 1800$

20. Use 3.14 for π.

1) What is the area of a circle having a radius of 1.2 meters? πr^2 $\quad 3.14 \times 1.2^2 = 4.52$

2) What is the diameter of a circle having a circumference of 60 feet? (Round off to nearest 10th.) $C = \pi d$ $\quad 60 = 3.14\, d$ $\quad d = 19.10$

3) What is the circumference of a circular pool having a radius of 20'? r=20 $\quad C = \pi d$ $\quad 3.14 \times 40 = 125.6$

4) The diameter of a half-dollar is 30 mm. What is its circumference? 30mm $\quad C = 3.14(30\text{ mm})$ $\quad 94.20$

5) Find the area of a half-dollar.
πr^2 $\quad 3.14(15^2) = 706.5$

21. 1) A stack of half-dollars is 50 mm high. What is its volume? (Remember the radius of a half-dollar is 15 mm.)

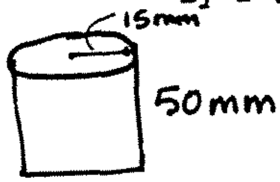
2) A tin box measures 5" by 3 $\frac{1}{2}$" by 2 $\frac{1}{2}$". What is its volume?

3) What is the volume of a smoke stack having a height of 200 feet and a radius of 2 feet?

4) A cup having a 3" diameter and 4" of height will hold how much coffee?

5) The measurements of a filing cabinet are 4' by 1 $\frac{1}{2}$' by 2'. What is its volume?

1.

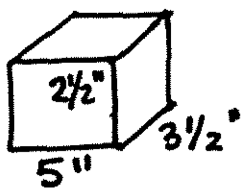


$V = \pi r^2 \times h$

$V = 3.14(15)^2 \times 50$

$V = 706.5 \times 50$ = 35,325 cubic meter

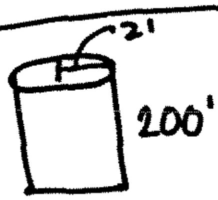
2.



V = l x w x h

V = 5 x 3.5 x 2.5 = 43.75 cubic inches

3.

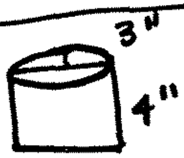


$V = \pi r^2 \times h$

$V = 3.14(2)^2 \times 200$

V = 2512 cubic ft.

4.



$V = \pi r^2 \times h$

$V = 3.14(1.5)^2 \times 4"$

V = 28.26 cubic inches

---

5. A = L x W x h   A = 1.5 x 2 x 4 = 12 cubic ft.

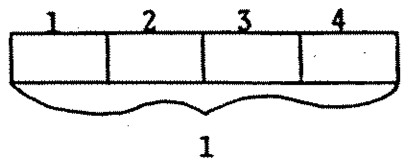
BASIC MATHEMATICS

FRACTIONS

I. Definition: A <u>fraction</u> represents part of a whole.

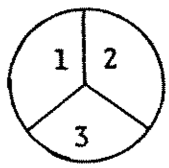
example:

a. $\frac{1}{4}$ means dividing one into 4 pieces, and taking one of them.



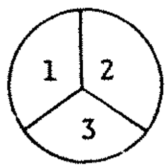
$\frac{1}{4}$

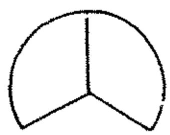
b. $\frac{1}{3}$ means dividing one into 3 pieces, and taking one of them.



$\frac{1}{3}$

c. $\frac{2}{3}$ means dividing one into 3 pieces, and taking 2 of them.





$\frac{2}{3}$

Terms: The <u>numerator</u> is the top number of the fraction.
The <u>denominator</u> is the bottom number of the fraction.

Any whole number (1, 2, 3, etc.) can be written as a fraction with a denominator of 1. $\frac{2}{1}$ means 2 wholes, which have not been divided into smaller units.

$\frac{5}{1} = 5$ $\frac{210}{1} = 210$

One can be represented by any fraction which has the numerator and denominator equal to the same number.

$1 = \frac{3}{3}$ $1 = \frac{5}{5}$

II. Multiplication

Multiply the numerators together and multiply the denominators together.

Symbols for multiplication are x, ·, ( ), ( )

example:

a. $\frac{3}{4} \times \frac{2}{5} = \frac{3}{4} \cdot \frac{2}{5} = \left(\frac{3}{4}\right)\left(\frac{2}{5}\right)$

$\frac{3}{4} \times \frac{2}{5} = \frac{3 \times 2}{4 \times 5} = \frac{6}{20}$

b. $\frac{5}{7} \times \frac{2}{3} = \frac{5 \times 2}{7 \times 3} = \frac{10}{21}$

Multiplication of fractions can be simplified by cancelling or reducing the fractions involved. Cancelling is reducing a fraction to its simplest terms. This is done by finding numbers which divide evenly into both the numerator and denominator.

example: $\frac{3}{6}$

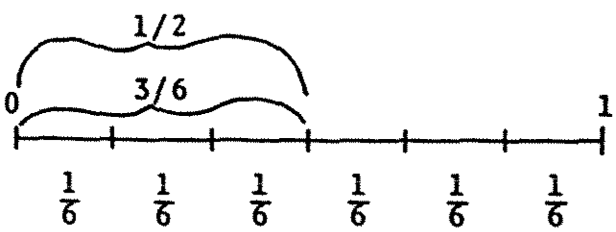
Both the top and bottom of this fraction can be divided evenly by 3.

$3 \div 3 = 1$ $\qquad$ $6 \div 3 = 2$

$\frac{3}{6} = \frac{3 \div 3}{6 \div 3} = \frac{1}{2}$

As long as the top and bottom are both divided by the same number, the value of the fraction remains the same.

We can see what has happened by using a number line:



When multiplying fractions we can also cancel the numerator of one fraction with the denominator of another, if both are divisible by a common number.

example: $\frac{3}{5} \times \frac{2}{9} = ?$

In this problem both fractions are in simplest terms, but the numerator of the first (3) and the denominator of the second (9) can both be divided by 3.

$$\frac{3 \div 3}{5} \times \frac{2}{9 \div 3} = \frac{1}{5} \times \frac{2}{3} = \frac{1 \times 2}{5 \times 3} = \frac{2}{15}$$

This can be written more simply as follows:

$$\frac{\cancel{3}1}{5} \times \frac{2}{\cancel{9}3} = \frac{2}{15}$$

When working with fractions (adding, subtracting, multiplying or dividing) final answers should always be reduced to lowest (simplest) terms.

Clues for reducing or cancelling fractions:

1. When the numerator and denominator both end in zero (0), they can be reduced by 10.

$$\frac{150}{40} = \frac{150 \div 10}{40 \div 10} = \frac{15}{4} \quad \text{or} \quad \frac{15\cancel{0}}{4\cancel{0}} = \frac{15}{4}$$

2. If both are even, they can be reduced by 2.

$$\frac{8}{26} = \frac{4}{13}$$

3. If both end in either a zero (0) or a 5, the fraction can be reduced by 5.

$$\frac{15}{20} = \frac{15 \div 5}{20 \div 5} = \frac{3}{4}$$

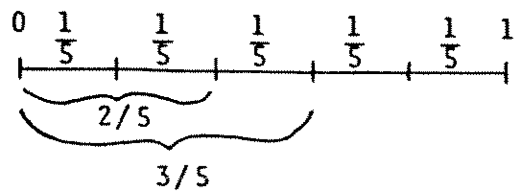
$$\frac{25}{35} = \frac{25 \div 5}{35 \div 5} = \frac{5}{7}$$

III. Addition

When adding fractions we can only add together parts of the same "size," meaning, those fractions with the same denominator. $\frac{1}{4}$ can be added directly to $\frac{3}{4}$, but not directly to $\frac{2}{3}$.

Fractions with the same denominator are added together by adding <u>only</u> the numerators and maintaining the same denominator.

example:



$$\frac{1}{5} + \frac{1}{5} = \frac{2}{5} \qquad \frac{2}{5} + \frac{1}{5} = \frac{3}{5}$$

What if the fractions do not have the same denominator? In this case, we convert them into new fractions which do have the same denominator. This denominator is called a common denominator.

A common denominator is a number both original denominators can be divided into evenly. A common denominator can be found for all fractional combinations.

1. One way to find a common denominator is to multiply the original denominators together.

example: $\frac{1}{3} + \frac{2}{5} = ?$

$3 \times 5 = 15$

Therefore, 15 would be the common denominator.

When converting the original fraction into a fraction with a common denominator, you must be sure to keep the value of the fraction the same. To do this, multiply both the numerator and denominator by the same number. Using the example above you proceed as follows:

$\frac{1x5}{3x5} = \frac{5}{15}$ $\qquad$ $\frac{2x3}{5x3} = \frac{6}{15}$

When you multiply both the top and bottom by the same number the value of the fraction does not change, because you are really multiplying by one. This problem can be finished by adding the numerators together, and maintaining the common denominator.

$\frac{5}{15} + \frac{6}{15} = \frac{5+6}{15} = \frac{11}{15}$

2. A smaller common denominator can often be found by mentally reviewing multiplication tables to find the smallest number both denominators divide into evenly.

example: $\frac{1}{6} + \frac{3}{8} = ?$

Both 6 and 8 will divide evenly into 24.

$24 \div 6 = 4$ $\qquad$ $24 \div 8 = 3$

Each denominator is multiplied by the number which will make it equal 24. The numerators are also multiplied by this number, so that the value of the fractions isn't changed.

$\frac{1x4}{6x4} = \frac{4}{24}$ $\qquad$ $\frac{3x3}{8x3} = \frac{9}{24}$

$\frac{4}{24} + \frac{9}{24} = \frac{4+9}{24} = \frac{13}{24}$

Using the last method (multiplying denominators) we would proceed as follows:

$$\frac{1 \times 8}{6 \times 8} = \frac{8}{48} \qquad \frac{3 \times 6}{8 \times 6} = \frac{18}{48}$$

$$\frac{8}{48} + \frac{18}{48} = \frac{8+18}{48} = \frac{26}{48}$$

$\frac{26}{48}$ can be reduced to $\frac{13}{24}$ by cancelling.

IV. Subtraction

Subtracting fractions is similar to adding fractions. First all of the fractions in the problem must be converted into new fractions with a common denominator. Once both fractions have a common denominator, the second numerator is subtracted from the first.

example: $\frac{3}{4} - \frac{1}{6} = ?$

The common denominator can be found 2 ways:

1. Multiply the denominators.

$4 \times 6 = 24$, the common denominator

$$\frac{3}{4} \times \frac{6}{6} = \frac{18}{24} \qquad \frac{1}{6} \times \frac{4}{4} = \frac{4}{24}$$

$$\frac{18}{24} - \frac{4}{24} = \frac{18-4}{24} = \frac{14}{24}$$

$\frac{14}{24}$ can be reduced to lower terms, or $\frac{7}{12}$.

2. Finding a smaller common denominator by finding a number both denominators will divide into evenly.

4 and 6 will both divide evenly into 12. 12 is smaller than 24, the denominator used in part 1. Since $12 \div 4 = 3$ and $12 \div 6 = 2$ we convert these fractions as follows:

$$\frac{3}{4} \times \frac{3}{3} = \frac{9}{12} \qquad \frac{1}{6} \times \frac{2}{2} = \frac{2}{12}$$

$$\frac{9}{12} - \frac{2}{12} = \frac{9-2}{12} = \frac{7}{12}$$

V. Division

In a division problem, the <u>divisor</u> is the number we are dividing by. It is written after the ÷ sign, or on the bottom of the slash.

example: $\frac{2}{3} \div \frac{1}{4}$ $\frac{1}{4}$ is the divisor

$$\frac{\frac{5}{8}}{\frac{1}{2}} = \frac{5}{8} \div \frac{1}{2}$$ $\frac{1}{2}$ is the divisor

Fractions are divided by inverting the divisor, and then multiplying. To invert the divisor, switch the numerator and denominator. Be sure to switch the division sign to a multiplication sign. Once the divisor has been inverted, multiply the 2 fractions together to get the answer.

example: $\frac{2}{3} \div \left(\frac{1}{4}\right) = ?$

$$\frac{2}{3} \times \frac{4}{1} = \frac{2\text{x}4}{3\text{x}1} = \frac{8}{3}$$

Example: $\frac{\frac{5}{8}}{\frac{1}{2}} = ?$

$$\frac{5}{8} \div \left(\frac{1}{2}\right) = \frac{5}{8} \times \frac{2}{1} = \frac{5\text{x}2}{8\text{x}1} = \frac{\cancel{10}5}{\cancel{8}4} = \frac{5}{4}$$

VI. Mixed Numbers

A <u>mixed number</u> has a whole number part and a fractional part (for example: $1\frac{1}{2}$, $20\frac{5}{9}$).

A mixed number can be represented entirely as a fraction. The new fraction will have the same denominator as the fractional part of the mixed number.

example: $2\frac{3}{4} = \frac{?}{4}$

Begin by looking at the whole number, $2 = \frac{?}{4}$. To find the numerator of this part, multiply the whole number by the denominator.

$$2 \times 4 = 8$$

Therefore, 2 represented in fractional form equals $\frac{8}{4}$.

$$2 = \frac{8}{4}$$

Now, add this to the fractional part of the mixed number.

$$2\frac{3}{4} = 2 + \frac{3}{4} = \frac{8}{4} + \frac{3}{4} = \frac{8+3}{4} = \frac{11}{4}$$

Fractions with the numerator greater than the denominator can be converted to mixed numbers. This is done by dividing the denominator into the numerator. The number of times the denominator goes into the numerator evenly is the whole number part of the mixed number. The remainder (the amount left over after dividing) is the new numerator of the fractional part. The denominator is the same.

example: $\frac{14}{3} = ?$

Three goes into 14 evenly four times. The remainder is 2.

$$\begin{array}{r@{\,}l} & 4 \\ 3 & \overline{)14} \\ & \underline{12} \\ & 2 \end{array}$$

$$\frac{14}{3} = 4\frac{2}{3}$$

VII. Multiplying Mixed Numbers and Fractions

example: $3\frac{1}{4} \times \frac{5}{8} = ?$

When multiplying with mixed numbers, first change the mixed numbers to a fraction.

$$3\frac{1}{4} = \frac{?}{4} \qquad 3 \times 4 = 12 \qquad 3 = \frac{12}{4}$$

$$3\frac{1}{4} = 3 + \frac{1}{4} = \frac{12}{4} + \frac{1}{4} = \frac{12+1}{4} = \frac{13}{4}$$

Once the mixed number has been converted to a fraction, this becomes a regular multiplication problem.

$$\frac{13}{4} \times \frac{5}{8} = \frac{13\text{x}5}{4\text{x}8} = \frac{65}{32}$$

This fraction should be represented as a mixed number.

$$\frac{65}{32} = ?$$

$$\begin{array}{r@{\,}l} & 2\frac{1}{32} \\ 32 & \overline{)65} \\ & \underline{64} \\ & 1 \end{array}$$

$$\frac{65}{32} = 2\frac{1}{32}$$

VIII. Division of Mixed Numbers

Division of mixed numbers is similar to multiplication. Convert the mixed numbers to fractions and then proceed as in a regular division problem with fractions.

example: $3\frac{2}{3} \div 2\frac{3}{4} = ?$

$3\frac{2}{3} = \frac{?}{3} \longrightarrow 3 + \frac{2}{3} = \frac{9}{3} + \frac{2}{3} = \frac{11}{3}$

$2\frac{3}{4} = \frac{?}{4} \longrightarrow 2 + \frac{3}{4} = \frac{8}{4} + \frac{3}{4} = \frac{11}{4}$

$\frac{11}{3} \div \left(\frac{11}{4}\right) = \frac{11}{3} \times \frac{4}{11} = \frac{\cancel{11}\times 4}{3\times\cancel{11}} = \frac{4}{3}$

$$\begin{array}{r} 1\frac{1}{3} \\ 3\overline{)4} \\ \underline{3} \\ 1 \end{array} \qquad \frac{4}{3} = 1\frac{1}{3}$$

IX. Addition and Subtraction of Mixed Numbers

When adding mixed numbers together add the whole numbers together and add the fractions together. It is not necessary to convert mixed numbers to fractions in order to add them. However, remember to find a common denominator for the fractional parts.

example: $4\frac{3}{8} + 6\frac{1}{12} = ?$

The common denominator is 24. Therefore,

$\frac{3}{8} \times \frac{3}{3} = \frac{9}{24}$ and $\frac{1}{12} \times \frac{2}{2} = \frac{2}{24}$

$$\begin{array}{rcr} 4\frac{3}{8} & = & 4\frac{9}{24} \\ +\ 6\frac{1}{12} & = & 6\frac{2}{24} \\ \hline ? & & 10\frac{11}{24} \end{array}$$

In a subtraction problem, follow the same procedure, subtracting whole numbers from each other and fractions from each other.

example: $6\frac{2}{5} - 2\frac{1}{7} = ?$

The common denominator is 35.

$$\frac{2}{5} \times \frac{7}{7} = \frac{14}{35} \qquad \frac{1}{7} \times \frac{5}{5} = \frac{5}{35}$$

$$\begin{array}{rcr} 6\,\frac{2}{5} & = & 6\,\frac{14}{35} \\ -\ 2\,\frac{1}{7} & = & 2\,\frac{5}{35} \\ \hline ? & & 4\,\frac{9}{35} \end{array}$$

DECIMALS

I. Definition: Decimal notation is a way of representing fractions which have denominators of 10 or a multiple of 10 (100, 1000, 10,000).

The prefix <u>deci</u> means 10. Our number system is of base 10. This means that $\overline{10}$ and multiples of 10 determine the <u>place values</u> in our number system. The following chart labels the place values in our base 10 system.

thousands	hundreds	tens	ones	tenths	hundredths	thousandths	ten thousandths	hundred thousandths
1	2	3	4.	5	6	7	8	9

245.62 is read "two hundred and forty-five and sixty two hundredths."

.7543 is read "seven thousand five hundred and forty-three ten thousandths."

Our money system illustrates how a base 10 system works. One cent is $\frac{1}{100}$ of a dollar. It can be written \$.01. When 10 one-cent pieces (pennies) are added together, we move into the next place value. This can be represented as $\frac{10}{100}$, $\frac{1}{10}$ of a dollar, or \$.10. One hundred pennies equal one dollar, which is written as $1.00.

II. Converting Decimals to Fractions

When converting decimals to fractions you must first figure out which place value represents the digit all the way to the right.

example: 2.541

One is the digit all the way to the right and the corresponding place value is thousandths.

This place value is then used as the denominator (1000). The numerator is simply the entire decimal number, with the decimal point removed (2541).

$$2.541 = \frac{2541}{1000}$$

This number can be converted to a mixed number, and will become $2\frac{541}{1000}$.

Note that the number of places to the right of the decimal point equals the <u>number of zeros</u> in the denominator.

example: $.03 = \frac{3}{100}$ (three hundredths)

example: $.7924 = \frac{7924 \div 4}{10{,}000 \div 4} = \frac{1981}{2500}$

III. Converting Fractions to Decimals

A fraction with the denominator equal to a multiple of 10 can be converted to a decimal very easily. First, count the number of zeros in the denominator. Then copy over the numerator. Starting from the digit all the way to the right, count digits, from <u>right to left</u> up to the number of zeros in the denominator. Place the decimal point to the <u>left</u> of this digit.

example: $\frac{4781}{100} = ?$

There are 2 zeros in the denominator. Count 2 digits from the right - 4781 - and place decimal to the left.

$$\begin{array}{c} 47\,8\,1 \\ \uparrow\uparrow \\ 2\,1 \end{array}$$

$$\frac{4781}{100} = 47.81$$

Note: If there are more zeros in the denominator than there are digits in the numerator, add as many zeros as necessary to the left of the numerator.

example: $\frac{34}{10{,}000} = ?$

There are 4 zeros in the denominator. Since there are only 2 digits in the numerator, add 2 zeros to the left - 34 = 0034. Now count 4 and place the decimal point to the left.

```
0034 = .0034
↑↑↑↑
4321
```

A fraction without a multiple of ten as the denominator can be converted to a decimal by dividing the denominator into the numerator. In most cases you will have to add the decimal point to the numerator and add zeros to the right of the decimal point in order to have the division come out evenly. You can add as many zeros as you need to the right of the decimal point. When doing the division, remember to carry the decimal point over the division sign.

example: $\frac{1}{8} = ?$ 1 = 1.000

```
     .125
8 ⟌1.000
    8
    20
    16
     40
     40
      0
```

$\frac{1}{8} = .125$

Sometimes this division will not come out evenly. In such cases either the division continues indefinitely in no specific pattern or the division shows a pattern of repeating digits (one or a series of digits).

<u>Repeating digits</u> are represented by a bar over those digits which repeat.

example: $\frac{2}{3} = .6\overline{6}$

```
    .66̄6
3 ⟌2.000
   1 8
     20
     18
      20
```

example: $\frac{2}{11} = .181\overline{8}$

```
      .1818̄
11 ⟌2.0000
    1 1
      90
      88
       20
       11
```

The following table lists some commonly used fractions and their decimal representation. Memorizing these conversions should speed up your work.

$\frac{1}{2} = .5$	$\frac{1}{10} = .1$
$\frac{1}{3} = .3\overline{3}$	$\frac{3}{4} = .75$
$\frac{1}{4} = .25$	$\frac{2}{3} = .6\overline{6}$
$\frac{1}{5} = .2$	$\frac{1}{25} = .04$
$\frac{1}{8} = .125$	$\frac{1}{100} = .01$

IV. Adding and Subtracting Decimals

Adding and subtracting decimals follows the same rules as adding and subtracting money.

example: If you have one dollar and you spend 73¢ how much money do you have left?

one dollar = \$1.00
73¢ = .73

$$\begin{array}{r} \$1.00 \\ -\ \underline{.73} \\ .27 \end{array}$$

The answer is .27 or 27¢.

The key to success is <u>lining up the decimal points</u>. Once the decimals are in line, work from <u>right</u> <u>to</u> <u>left</u> down the columns.

example: .532 + .219 + .90 + .0002 = ?

Line each decimal up as follows:

$$\begin{array}{r} .532 \\ .219 \\ .90 \\ +\ \underline{.0002} \\ 1.6512 \end{array}$$

Start from the right-most column and add.

If a whole number is in the problem, remember that you can change it to a decimal by adding a decimal point to the <u>right</u> end of the number.

example: .25 + .017 + 15.1 + 7 = ?

7 = 7.

$$
\begin{array}{r}
.25 \\
.017 \\
15.1 \\
+\ 7. \\
\hline
22.367
\end{array}
$$

The same rules apply to subtraction.

example: .103 - .091 = ?

$$
\begin{array}{r}
.103 \\
-\ .091 \\
\hline
.012
\end{array}
$$

Remember to line up the decimal point and start subtracting from the right-most column.

Zeros added to the right of the last digit in a decimal will not change the value of the decimal. Adding zeros may be helpful in visualizing the computations.

example:

$$
\begin{array}{r}
.024 \\
+\ .1 \\
\hline
?
\end{array}
\longrightarrow
\begin{array}{r}
.024 \\
.100 \\
\hline
.124
\end{array}
$$

example:

$$
\begin{array}{r}
.2 \\
-\ .05 \\
\hline
?
\end{array}
\longrightarrow
\begin{array}{r}
.20 \\
-\ .05 \\
\hline
.15
\end{array}
$$

Note: Zeros to the left of the digits and between the digits and the decimal point are place holders and cannot be removed without changing the value of the fraction. The decimal number .05 does not equal .5 or .005. However, .05 does equal .050 and .0500.

$$.05 \neq .5 \neq .005$$
$$.05 = .050 = .0500$$

When subtracting, "borrowing" one from the column to the left can be done when necessary, the same way it is done when subtracting whole numbers.

example:

$$
\begin{array}{r}
.704 \\
-\ .39 \\
\hline
?
\end{array}
\qquad
\begin{array}{r}
.704 \\
.390 \\
\hline
?
\end{array}
\qquad
\begin{array}{r}
.\overset{6}{\cancel{7}}\overset{1}{0}4 \\
.390 \\
\hline
.314
\end{array}
$$

Remember to check subtraction by adding the answer to the amount subtracted. These 2 should equal the first number. To check the answer in the problem above we would add .314 + .390.

$$
\begin{array}{r}
.314 \\
+\ .390 \\
\hline
.704
\end{array}
$$

The result, .704, equals the top number in the problem above, so we see the subtraction was correct.

V. Multiplying Decimals

To multiply two decimal numbers together, begin by multiplying the numbers and ignoring the decimal points. Then count the total number of places (digits) to the right of the decimal point in both original numbers. This will be the number of places after the decimal point in the answer. Starting from the digit all the way to the right, count this number of places to the left, and place the decimal point there.

example: $1.2 \times .09 = ?$

$$\begin{array}{r} 1.2 \\ \times\ \underline{.09} \\ ? \end{array} \rightarrow \begin{array}{r} \overset{1}{1}2 \\ \underline{09} \\ 108 \end{array}$$

Since there is 1 digit to the right of the first decimal and 2 to the right of the second, we count back 3 digits in the answer.

$108 = .108$

If there are not enough digits in the answer, add zeros to the left (to fill in the place values).

example: $.0002 \times .03 = ?$

$$\begin{array}{r} .0002 \\ \times\ \underline{.03} \\ ? \end{array} \rightarrow \begin{array}{r} 0002 \\ \underline{03} \\ 6 \end{array}$$

Since we have to count 6 places, add 5 zeros.

.000006 is the answer.

VI. Dividing Decimals

To divide decimals when the divisor (the number you are dividing by) is not a decimal, carry out the division as usual without removing the decimal point from the dividend (the number being divided). The decimal point is placed in the answer (the quotient) directly above the decimal in the dividend.

example: $.4963 \div 7 = ?$

First set up the problem for division. Place the decimal directly above the decimal in the dividend.

$$\begin{array}{r} . \\ 7 \overline{\big) .4963} \end{array}$$

Since 7 divides into 49, not 4, place a zero in the first place to the right of the decimal as a place holder. Now divide, as usual.

$$\begin{array}{r} .0709 \\ 7 \overline{\big) .4963} \\ \underline{49} \\ 063 \\ \underline{63} \\ 0 \end{array}$$

To divide when the divisor is a decimal, move the decimal point in the divisor to the right end of the number. Now move the decimal point in the dividend the same number of places to the right. Put the decimal for the quotient directly above the newly placed decimal in the dividend.

example: $361.6 \div .08 = ?$

```
                           4520.
.08 ⟌361.60.        08 ⟌36160.
                         32
                          41
                          40
                           16
                           16
                            00
```

Move the decimal in the divisor (.08) two places to the right. Do the same in the dividend (361.6). You will have to add a zero to the <u>right</u> of the 6 in order to move this decimal point 2 <u>places</u>. Place the decimal in the quotient and divide. Do <u>not</u> forget to include the zero between the 2 and <u>decimal point</u> in the answer.

example: $.04745 \div .13 = ?$

```
                            .365
.13  ⟌.04745        13 ⟌4.745
                         3 9
                           84
                           78
                            65
                            65
                             0
```

PERCENT

I. Definition: A <u>percent</u> is a fraction with a denominator of 100. Because they <u>are used</u> so frequently, hundredths have been given this special notation, percent, written %.

example: 20% (20 percent) means 20 hundredths or $\frac{20}{100}$.

325% (325 percent) means 325 hundredths or $\frac{325}{100}$.

.5% (.5 percent) means .5 hundredths or $\frac{.5}{100}$.

II. Changing Percents to Decimals

Because percents are fractions with denominators of 100 (which is a multiple of 10), they can easily be written in decimal notation. Do this by moving the decimal point two places to the <u>left</u>. If no decimal point is given, remember to write one <u>in to</u> the <u>right</u> of the last digit.

example: $5\% = 5.\% = \frac{5}{100} = .05$

$.6\% = \frac{.6}{100} = .006$

$370\% = \frac{370}{100} = 3.70$

Decimals can be converted to percentages by doing the opposite, moving the decimal point 2 places to the right.

example: $.71 = 71\%$

If you have trouble remembering which way to move the decimal point, thinking of your sales tax can be helpful. You can write on your scrap paper, for example:

$7\% = .07$ $.07 = 7\%$

(The tax is 7 cents on every dollar spent). Then, when a percent like .035% comes along, and you need to convert it to a decimal, it becomes easier. Looking at the tax, we see that 7% = .07. The decimal point was moved two places to the left, so we would do the same to .035%.

$.035\% = .00035$

If you had a decimal like .0068, and needed to convert it to a percent, it would become easier by contrasting it to .07 = 7%. Here the decimal point was moved two places to the right, so we would do the same:

$.0068 = .68\%$

Before we can add, subtract, multiply or divide with percents, they must be rewritten in fractional or decimal notation.

Percentages, like fractions, express part of something. 100% represents a whole quantity. 100% of the students in the class means all the students. 100% of any number is itself. Think of 100% as "all."

example: 100% of 5 is 5.
83 is 100% of 83.

The relationship between two numbers is often shown by expressing one as so many percent of the other.

example: 10 is 50% of 20

Think of the word "is" as an equals sign (=) and the word "of" as a multiplication sign (x). Now we can transform the above statement into a math equation.

10 is 50% of 20

$$10 = 50\% \times 20$$

$$10 = \frac{50}{100} \times 20$$

$$10 = \frac{1000}{100}$$

or

$$10 = .50 \text{ of } 20$$

$$10 = .50 \times 20$$

$$10 = 10$$

example: 25% of 8 is 2

Translating that into a math equation we see that:

$$25\% \times 8 = 2$$

$$\frac{25}{100} \times 8 = 2$$

$$\frac{200}{100} = 2$$

or

$$.25 \times 8 = 2.0$$

Using this example we see that a <u>percent</u> (25%) of a <u>base</u> number (8) is a <u>percentage</u> (2) of the base. The standard equation is:

Percent x Base = Percentage

Given values for any <u>two</u> elements of this equation, we can find the third.

example: Find 5% of 250.

We are given the percent and the base and must find the percentage.

$$5\% \times 250 = \text{percentage}$$

$$\frac{5}{100} \times \frac{250}{1} = \frac{1250}{100} = 12.5$$

or

$$.05 \times 250 = 12.5$$

Therefore, 12.5 is 5% of 250.

example: 30 is what percent of 50?

$$30 = \text{percent} \times 50$$

When you are looking for the percent and your equation, like the one above, has a number on either side of the equal sign, take the number that the percent is multiplied by and divide it into the number on the other side. (Or, looking at it another way, move the number to the other side of the equal sign and change the multiplication sign to a division sign.)

$$\frac{30}{50} = \text{percent} \times 50$$

$$\frac{30}{50} = \text{percent}$$

To change $\frac{30}{50}$ to a fraction with a denominator of 100, multiply both the numerator and denominator by 2.

$$\frac{30 \times 2}{50 \times 2} = \frac{60}{100}$$

$$\text{percent} = \frac{60}{100} = 60\%$$

example: 12 is 60% of what number?

In this problem we are asked to find the <u>base</u> number.

$$12 = 60\% \times \text{base}$$

$$12 = \frac{60}{100} \times \text{base}$$

We solve this, like finding the percent in the problem above, by moving $\frac{60}{100}$ to the left side and dividing 12 by it.

$$12 \div \frac{60}{100} = \text{base}$$

Remember that to divide by a fraction you invert the top and bottom and then multiply.

$$12 \div \left(\frac{60}{100}\right) = \text{base}$$

$$\overset{1}{\cancel{12}} \times \frac{100}{\cancel{60}_{5}} = \text{base}$$

$$\frac{100}{5} = 20 = \text{base}$$

We use percents every day to describe many situations. Percents are common in word problems too, so some have been included.

example: Forty-five of the 60 employees of Department A attended the annual picnic. What percentage of the department's employees attended?

We are given:

Total number of employees = 60 = base number

Part of total = 45 = percentage

to find the <u>percent</u>.

This problem can be reworded as follows:

45 is what percent of 60?

$$\frac{45}{60} = \text{percent} \times 60$$

$$\frac{45}{60} = \text{percent}$$

Convert $\frac{45}{60}$ to a decimal be dividing the numerator by the denominator.

$$\begin{array}{r} .75 \\ 60 \overline{)45.00} \\ 42\ 0 \\ \hline 3\ 00 \end{array}$$

Convert the decimal .75 to a percent by moving the decimal point 2 places to the <u>right</u>.

.75 = 75%

III. Percent Increase and Decrease

Percents are commonly used to compare changes in quantity. For example, the couch you want may be reduced by 15% and your car insurance increased by 5%. The changes are expressed as a percentage of the <u>original quantity</u> (base number). We can also see from such comparisons that a $5 increase in a $10 book (to $15) is a lot more significant than a $5 increase in a $100 bicycle (to $105).

Using our knowledge of percents, we can calculate that:

$5 is 50% of $10, but

$5 is 5% of $100.

The book's price increased by 50% while the bicycle's price increased by only 5%.

The new prices of the book and bicycle are found by adding the original price plus the increase. Therefore,

New amount = Original + Increase

15 =	10	+	5
105 =	100	+	5

Likewise, when a price is reduced, the new amount is found by subtracting the decrease from the original price.

New amount = Original - Decrease

Let's look at other examples.

Percent Decrease

A couch originally selling for $480 is now reduced by 15%. How much is the couch selling for on sale?

We are looking for the new price. We know that: New price = Original - Decrease. Since we have the original price, we must first find the decrease.

15% of $480 is the decrease

$.15 \times 480 = 72$, or

$\frac{15}{10\cancel{0}} \times 48\cancel{0}$ = decrease

$\frac{720}{10}$ = decrease

Going back to our formula:

New price = Original - Decrease

New price = $480 - 72

New price = $408

We can write both steps in one equation:

New price = Original - (% x Original)

New price = $480 - $(\frac{15}{100} \times 480)$

When working with multiplication or division and addition or subtraction in the same equation, always do the multiplication or division first.

Percent Increase

Your car insurance costs $220 each six months. With your last bill, you were notified of a 5% increase in cost. How much will your next insurance bill be?

We are given the original number and the percent change. First, find the amount of the increase:

5% of $220 is the increase

5% x $220 = increase

$\frac{5}{10\cancel{0}} \times 22\cancel{0}$ = increase

$\frac{110}{10}$ = 11 = increase

Then use the formula:

New amount = Original + Increase

New amount = 220 + 11

New amount = 231

This can be done in one equation as follows:

New amount = Original + (% x Original)

New amount = 220 + (5% x 220)

New amount = 220 + ($\frac{5}{100}$ x 220)

231 = 220 + 11

Note: It is important to realize that the change is expressed as a percentage of the <u>original quantity</u>, not of the new value.

220 increased by 5% is 231. The change is 11 and 11 is 5% of 220. 11 is <u>not</u> 5% of 231.

Likewise, from our first problem, the couch was reduced by 15%. $480 decreased by 15% is $408. The decrease is $72. 72 is 15% of 480. 72 is <u>not</u> 15% of 408.

Another way to look at percent increase or decrease is as follows:

The original price of the couch ($480) is 100%. When the couch is reduced by 15%, the new price will be 100% - 15% or 85%. If you look at the problem this way, the new price is 85% of the original price.

New price = 85% of \$480

New price = .85 x 480

New price = \$408

Likewise, your original car insurance (\$220) is 100%. When it increases 5%, your new insurance will be 100% + 5%, or 105%. The new insurance is 105% of the original insurance.

New insurance = 105% of \$220

New insurance = 1.05 x 220

New insurance = \$231

When working with prices, the terms "mark-up" and "mark-down" can be used to describe percent increase and decrease. Mark-up means the price is increased by a given percent, while mark-down means it has been decreased by a given percent.

example: A portable radio is marked down 20% to a cost of \$64. What was the original cost of the radio?

We solve this like a percent decrease problem. We don't know the original price, but we <u>do</u> know the <u>new</u> price and the percent decrease. If the original price is 100%, the new price is 100% - 20%, or 80% of the original. This problem can be rewritten: The new price is 80% of the original price.

$64 = 80\% \times \text{original}$

$64 = \frac{80}{100} \times \text{original}$

$64 \div \frac{80}{100} = \text{original}$

$\overset{8}{\cancel{64}} \times \frac{10\cancel{0}}{8\cancel{0}} = \$80 = \text{original price, or}$

$64 \div .80 = 80$

It is possible to figure out the percent increase or decrease if the original amount and the new amount are given.

example: Joan's electric bill was \$56 last month and \$60 this month. By what percent did her bill increase?

New amount = Original + Increase

\$60 = \$56 + increase

60 - 56 = increase

\$4 = increase

$$\text{increase} = \% \times \text{original}$$

$$4 = \% \times \$56$$

$$\frac{\cancel{4}1}{\cancel{\$}\cancel{5}\cancel{6}14} = \%$$

$$\frac{1}{14} = \%$$

$$\begin{array}{r} .0714 \\ 14\,\overline{)\,1.0000} \\ \underline{98} \\ 20 \\ \underline{14} \\ 60 \\ \underline{56} \\ 4 \end{array} = .07\%$$

There is another way to do these type of problems, which are often found on tables on exams, that will always work, and is easier for some people. To determine the percent increase or decrease, FIRST FIND THE DIFFERENCE BETWEEN THE TWO NUMBERS BEING CONSIDERED, AND THEN DIVIDE THIS DIFFERENCE BY THE ORIGINAL NUMBER, THE NUMBER THAT CHRONOLOGICALLY CAME FIRST. In this case, the difference is $4. This difference should be divided by the original number, last month's $56 figure.

$$\frac{4}{56} = .07 = 7\% \text{ increase}$$

These type of questions are reviewed in detail, with plenty of practice questions, in Booklet #3, Understanding and Interpreting Tabular Material.

RATIOS

A ratio is a comparison of one number to another. A ratio between two numbers is expressed by putting a colon (:) between them like this:

Ratio of women to men in the office is 1 to 2.

women in office: # men in office = 1:2

Ratios can be written as fractions:

women in office: # men in office = 1:2 = $\frac{1}{2}$

Ratios can be reduced to simplest form by writing them as fractions and simplifying. Given the ratio as 3:6 we can re-write it as $\frac{3}{6}$ and then reduce it to $\frac{1}{2}$.

example: Reduce 5:10.

$$5:10 = \frac{5}{10} = \frac{\cancel{5}1}{\cancel{10}2} = \frac{1}{2}$$

example: Reduce 7:28.

$$7:28 = \frac{7}{28} = \frac{\cancel{7}1}{\cancel{28}4} = \frac{1}{4}$$

Sometimes the term "ratio" will be used in a word problem, but often the language will suggest a comparison and you must recognize it as a ratio problem. Look for key words such as "compared to," "is to," "out of," "relationship between." Working with ratios involves working with fractions:

1 woman <u>out of</u> 20 = 1:20 or $\frac{1}{20}$

3 <u>is to</u> 9 = 3:9 or $\frac{3}{9}$

When asked to work with ratios, you are usually given one comparison or ratio (or fraction) and asked to find an equivalent one.

example: There are 340 employees in an office. One out of every 20 workers will take a vacation in June. How many workers will take a vacation in June?

You are given the ratio 1 to 20 or 1:20, $\frac{1}{20}$. You are asked how many of the 340 workers will take a vacation in June. To set up an equivalent fraction, first note what the comparison is and as a safeguard, write it out next to your work. (The line of the fraction can be substituted for any of the key words listed above.) The comparison is <u>workers taking a vacation</u> to <u>all workers</u>.

$$\text{(compared to) } \frac{\text{workers on vacation}}{\text{all workers}} : \frac{1}{20} = \frac{?}{340}$$

Another way to read it would be to say, "1 is to 20 as what is to 340?"

1:20 = ?:340

$$\frac{1}{20} \text{ of } 340 = \frac{1}{20} \times 340 = \frac{34\cancel{0}}{2\cancel{0}} = 17, \text{ or}$$

$$\frac{1}{20} = .05 \longrightarrow .05 \times 340 = 17$$

example: How far will a plane travel in 15 hours if it travels 1500 miles in 5 hours and continues at the same rate of speed?

The comparison is the number of miles to the number of hours.

$$\frac{\text{miles}}{\text{hours}} = \frac{1500}{5} = \frac{?}{5}$$

$$\frac{\text{miles}}{\text{hours}} = \frac{1500}{5} = \frac{?}{15}$$

We are looking for a number that has the same relationship to 15 that 1500 has to 5. Because we are working with equivalent fractions, this number will also have the same relationship to 1500 that 5 has to 15. Using this second piece of information, we can find the missing number.

We know that 3 x <u>5</u> = <u>15</u>.

We multiply 5 by 3 to get 15.

The missing number is in the same relationship to 1500 as 5 is to 15. Therefore:

$$3 \times 1500 = ?$$

$$3 \times 1500 = 4500$$

In review we see that:

$$\frac{3 \times 1500}{3 \times 5} = \frac{4500}{15}$$

When the relationships between the numbers is not as familiar, we follow the procedure shown below:

$$\frac{1500}{5} = \frac{?}{15}$$

1. First cross-multiply.

$$\frac{1500}{5} = \frac{?}{15} \longrightarrow 1500 \times 15 = 22,500$$

2. Now divide this total by the third number, 5 in this case.

$$22,500 \div 5 = 4500$$

3. Then check by comparing the fractions (reducing will help in checking the answers).

$$\frac{\overset{300}{\cancel{1500}}}{\cancel{5}\,1} = \frac{\overset{300}{\cancel{4500}}}{\cancel{15}\,1}$$

example: $\frac{3}{?} = \frac{2}{14}$

In this problem, cross-multiply 3 x 14 to get 42. Then divide 42 by the third number, in this case 2, to get 21.

$$42 \div 2 = 21$$

Now check for equivalence:

$$\frac{\cancel{3}1}{\cancel{21}7} = \frac{\cancel{2}1}{\cancel{14}7}$$

$$\frac{1}{7} = \frac{1}{7}$$

The ratios we have worked with so far compared a "part" to a "whole." In this next section we will work with ratios which compare parts to parts, as well as parts to the whole.

It is important to determine which type of ratio you are working with in order to set up the correct equation.

example: For every five permanent workers in the office, there is one provisional employee. In an office with 60 employees, how many are provisional?

We are given a comparison of permanent workers to provisional workers, but the question asks you to compare provisional workers to all workers in the office.

We are comparing:

$$\frac{\text{provisional workers}}{\text{permanent workers}} = \frac{1}{5}$$

We want to compare:

$$\frac{\text{provisional workers}}{\text{all workers}}$$

Since there are 60 workers in the office all together, think of $\frac{1}{5}$ as the fraction, reduced to lowest terms, which represents 60 divided into parts in a relationship of 1:5. Then the total number of workers in the office is represented by adding these parts:

# provision workers	+	permanent workers	=
1	+	5	= 6

Therefore:

$$\frac{\text{provisional workers}}{\text{all workers}} = \frac{1}{5+1} = \frac{1}{6}$$

We can set this equal to:

$$\frac{1}{6} = \frac{?}{60}$$

Now follow the usual procedure to solve:

1. First cross-multiply.

$$\frac{1}{6} = \frac{?}{60} \longrightarrow 1 \times 60 = 60$$

2. Then divide by the third number.

$$60 \div 6 = 10$$

3. Check by reducing.

$$\frac{1}{6} = \frac{\cancel{10}1}{\cancel{60}6}$$

Another way to do this is to remember that, for this type of ratio problem, it can be solved by always adding the "parts" involved, in this case 1 + 5, and dividing the resulting number, 6, into the total of people given, 60. This will give you the value of <u>each part</u>.

$$60 \div 6 = 10$$

If the problem was slightly changed, and there were 4 permanent employees for every 2 provisional, adding them you'd get 6; dividing it into 60 you'd get 10, but that means <u>each part</u> was worth 10. Since the ratio is now 2 provisional workers to 4 permanent, and now 2 x 10 = 20, there would be 20 provisional workers. Booklet #2, Arithmetic Reasoning, has more examples of this type of question.

Let's re-word the problem again:

> For every five permanent workers in the office there is one provisional employee. If there are 50 permanent workers, how many total employees in the office are there?

Again, we have to translate a part-to-part ratio to a part-to-whole ratio. If there are five permanent workers to one provisional worker $\left(\frac{5}{1}\right)$ then what is the ratio of permanent workers to all employees? Add the parts (permanent + temporary = 5 + 1 = 6) to see that 5 out of 6 $\left(\frac{5}{6}\right)$ employees in the office are permanent.

The comparison we are asked to draw is permanent workers to total employees:

$$\frac{\text{permanent}}{\text{total}} \qquad \frac{50}{?} = \frac{5}{6}$$

Cross-multiply $\frac{50}{?} = \frac{5}{6}$ → 50 x 6 = 300, and then divide by 5 → 300 ÷ 5 = 60 to reach the answer: <u>60</u>.

Or, you could say, "What number is in the same relationship to 50 as 5 is to 6," and figure it out by determining what number 50 is $\frac{5}{6}$ of.

$$50 \div \frac{5}{6} = 50 \times \frac{6}{5} = \frac{300}{5} = 60$$

There is a lot of work with ratios, percents, decimals, and fractions in Booklets 2 and 3, Arithmetic Reasoning and Understanding and Interpreting Tabular Material. This booklet is intended to give you a refresher of the basics, so you'll be able to do the word problems and tabular questions more easily.

GOOD LUCK!