BASIC MATH REVIEW FOR THE ENGINEERING TECHNICIAN EXAM.

ALGEBRA:

POLYNOMIALS

$$(a+b)(a-b) = a^2 - b^2$$

$$(a \pm b)^2 = a^2 \pm 2ab + b^2$$

$$(a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$$

$$(a^n + b^n) = (a+b)(a^{n-1} - a^{n-2}b + a^{n-3}b^2 - \ldots + b^{n-1})$$

ROOTS OF QUADRATIC EQUATIONS

$$ax^2 + bx + c = 0$$

$$x_1, x_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x_1 + x_2 = -b/a \qquad x_1 x_2 = \frac{c}{a}$$

BASIC MATH REVIEW FOR THE
ENGINEERING TECHNICIAN EXAM

RULES FOR EXPONENTS AND RADICALS

$$b^0 = 1 \qquad b \neq 0$$

$$b^1 = b$$

$$b^{-n} = \frac{1}{b^n} = \left(\frac{1}{b}\right)^n \qquad b \neq 0$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \qquad b \neq 0$$

$$(ab)^n = a^n b^n$$

$$b^{m/n} = \sqrt[n]{b^m} = \left(\sqrt[n]{b}\right)^m$$

$$b^m b^n = b^{m+n}$$

$$\frac{b^m}{b^n} = b^{m-n}$$

$$\sqrt[n]{b} = b^{\frac{1}{n}}$$

BASIC MATH REVIEW FOR THE ENGINEERING TECHNICIAN EXAM

LOGARITHMS

$$\log_b (b) = 1$$

$$\mathrm{Log}_b (1) = 0$$

$$\mathrm{Log}_b (b^N) = N$$

$$\mathrm{Log} (\sqrt[N]{X} = \mathrm{Log} (X^{\frac{1}{N}}) = \frac{\log (x)}{N}$$

$$\mathrm{Log} (xy) = \log (x) + \log (y)$$

$$\mathrm{Log} \frac{x}{y} = \log(x) - \log(y)$$

Find CHARACTERISTICS & MANTISSAS

1) 40.60 = 1.60853
2) 9.28 = 0.96755
3) .0071 = -2.14874
4) 4070.9 = 3.60969

BASIC MATH FOR THE
ENGINEERING TECHNICIAN EXAM

SIMULTANEOUS LINEAR EQUATIONS

$2x + 3y = 24$ EQUATION 1

$3x + 4y = 33$ EQUATION 2

A) From EQUATION 1 solve for Variable X

$$x = \frac{24 - 3y}{2} = 12 - 1.5y$$

IN EQUATION 2
substitute $12 - 1.5y$ INTO EQUATION 2 where ever x is

$$3(12 - 1.5y) + 4y = 33$$

$$36 - 4.5y + 4y = 33$$

$$-.5y = -3$$

$$\left.\begin{array}{l} y = 6 \\ x = 3 \end{array}\right\} \text{ANSWER}$$

BASIC MATH FOR THE ENGINEERING TECHNICIAN EXAM

LIMITS:

A LIMIT IS THE VALUE A FUNCTION APPROACHES WHEN AN INDEPENDENT VARIABLE APPROACHES A TARGET VALUE.

$y = x^2$ LIMIT AS X APPROACHES 6

$$L = \lim_{x \to 6} x^2 = 36$$

$$\lim_{x \to a} x = a$$

$$\lim_{x \to a} (mx + b) = ma + b$$

$$\lim_{x \to 0} \text{any } x = 0$$

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \to 0} \cos x = 1$$

BASIC MATH FOR THE
ENGINEERING TECHNICIAN EXAM

LIMITS: EVALUATE the following Limits

$$\lim_{x \to 3} \frac{x^3 - 27}{x^2 - 9} = \frac{(x-3)(x^2+3x+9)}{(x-3)(x+3)}$$

$$= \lim_{x \to 3} \frac{3^2 + (3)(3) + 9}{3+3} = \frac{9}{2} \quad \text{ANS.}$$

- LINEAR ALGEBRA -
GEOMETRIC FORMULAS

4 GEOMETRIC FORMULAS

RECTANGLE OF LENGTH b AND WIDTH a

4.1 Area $= ab$

4.2 Perimeter $= 2a + 2b$

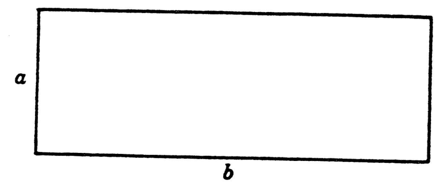


Fig. 4-1

PARALLELOGRAM OF ALTITUDE h AND BASE b

4.3 Area $= bh = ab \sin\theta$

4.4 Perimeter $= 2a + 2b$

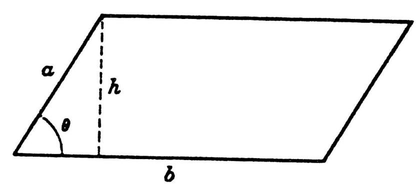


Fig. 4-2

TRIANGLE OF ALTITUDE h AND BASE b

4.5 Area $= \frac{1}{2}bh = \frac{1}{2}ab \sin\theta$

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

where $s = \frac{1}{2}(a+b+c) =$ semiperimeter

4.6 Perimeter $= a + b + c$

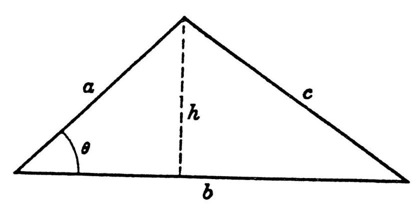


Fig. 4-3

TRAPEZOID OF ALTITUDE h AND PARALLEL SIDES a AND b

4.7 Area $= \frac{1}{2}h(a+b)$

4.8 Perimeter $= a + b + h\left(\frac{1}{\sin\theta} + \frac{1}{\sin\phi}\right)$

$$= a + b + h(\csc\theta + \csc\phi)$$

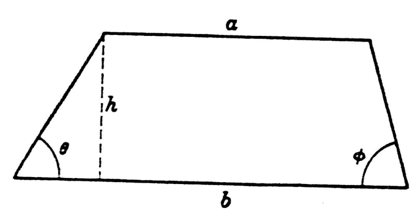


Fig. 4-4

REGULAR POLYGON OF n SIDES EACH OF LENGTH b

4.9 Area $= \frac{1}{4}nb^2 \cot\frac{\pi}{n} = \frac{1}{4}nb^2 \frac{\cos(\pi/n)}{\sin(\pi/n)}$

4.10 Perimeter $= nb$

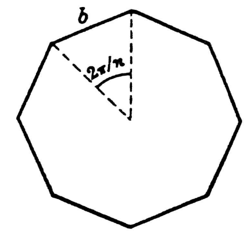


Fig. 4-5

CIRCLE OF RADIUS r

4.11 Area $= \pi r^2$

4.12 Perimeter $= 2\pi r$

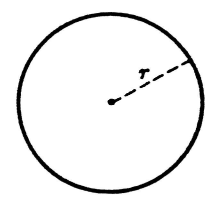


Fig. 4-6

SECTOR OF CIRCLE OF RADIUS r

4.13 Area $= \frac{1}{2}r^2\theta$ [θ in radians]

4.14 Arc length $s = r\theta$

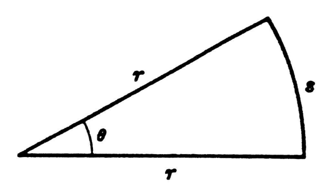


Fig. 4-7

RADIUS OF CIRCLE INSCRIBED IN A TRIANGLE OF SIDES a, b, c

4.15
$$r = \frac{\sqrt{s(s-a)(s-b)(s-c)}}{s}$$

where $s = \frac{1}{2}(a+b+c) =$ semiperimeter

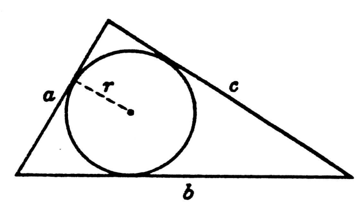


Fig. 4-8

RADIUS OF CIRCLE CIRCUMSCRIBING A TRIANGLE OF SIDES a, b, c

4.16
$$R = \frac{abc}{4\sqrt{s(s-a)(s-b)(s-c)}}$$

where $s = \frac{1}{2}(a+b+c) =$ semiperimeter

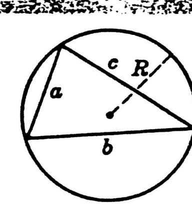


Fig. 4-9

REGULAR POLYGON OF n SIDES INSCRIBED IN CIRCLE OF RADIUS r

4.17 Area $= \frac{1}{2}nr^2 \sin\frac{2\pi}{n} = \frac{1}{2}nr^2 \sin\frac{360^\circ}{n}$

4.18 Perimeter $= 2nr \sin\frac{\pi}{n} = 2nr \sin\frac{180^\circ}{n}$

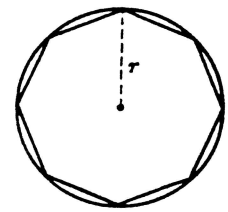


Fig. 4-10

REGULAR POLYGON OF n SIDES CIRCUMSCRIBING A CIRCLE OF RADIUS r

4.19 Area $= nr^2 \tan\frac{\pi}{n} = nr^2 \tan\frac{180^\circ}{n}$

4.20 Perimeter $= 2nr \tan\frac{\pi}{n} = 2nr \tan\frac{180^\circ}{n}$

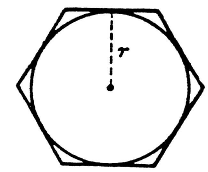


Fig. 4-11

SEGMENT OF CIRCLE OF RADIUS r

4.21 Area of shaded part $= \frac{1}{2}r^2(\theta - \sin\theta)$

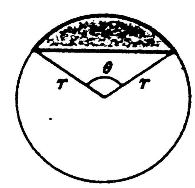


Fig. 4-12

ELLIPSE OF SEMI-MAJOR AXIS a AND SEMI-MINOR AXIS b

4.22 Area $= \pi ab$

4.23 Perimeter $= 4a\int_0^{\pi/2} \sqrt{1 - k^2 \sin^2\theta}\, d\theta$

$= 2\pi\sqrt{\frac{1}{2}(a^2 + b^2)}$ [approximately]

where $k = \sqrt{a^2 - b^2}/a$. See page 254 for numerical tables.

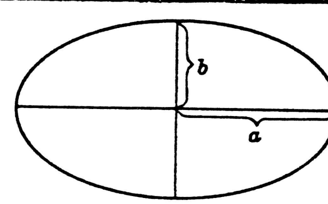


Fig. 4-13

SEGMENT OF A PARABOLA

4.24 Area $= \frac{2}{3}ab$

4.25 Arc length $ABC = \frac{1}{2}\sqrt{b^2 + 16a^2} + \frac{b^2}{8a}\ln\left(\frac{4a + \sqrt{b^2 + 16a^2}}{b}\right)$

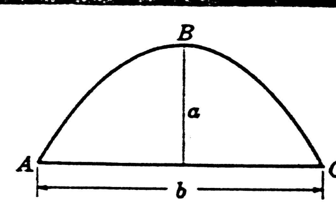


Fig. 4-14

RECTANGULAR PARALLELEPIPED OF LENGTH a, HEIGHT l, WIDTH c

4.26 Volume $= abc$

4.27 Surface area $= 2(ab + ac + bc)$

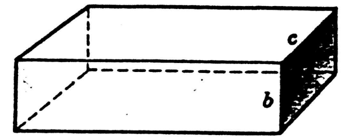


Fig. 4-15

PARALLELEPIPED OF CROSS-SECTIONAL AREA A AND HEIGHT h

4.28 Volume $= Ah = abc \sin\theta$

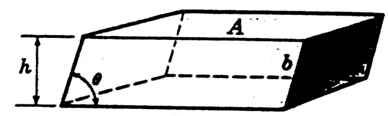


Fig. 4-16

SPHERE OF RADIUS r

4.29 Volume $= \frac{4}{3}\pi r^3$

4.30 Surface area $= 4\pi r^2$

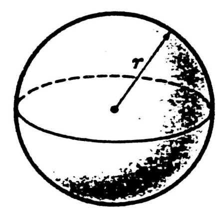


Fig. 4-17

RIGHT CIRCULAR CYLINDER OF RADIUS r AND HEIGHT h

4.31 Volume $= \pi r^2 h$

4.32 Lateral surface area $= 2\pi r h$

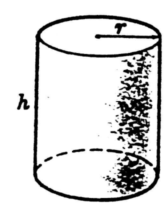


Fig. 4-18

CIRCULAR CYLINDER OF RADIUS r AND SLANT HEIGHT l

4.33 Volume $= \pi r^2 h = \pi r^2 l \sin\theta$

4.34 Lateral surface area $= 2\pi r l = \dfrac{2\pi r h}{\sin\theta} = 2\pi r h \csc\theta$

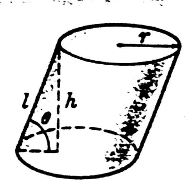


Fig. 4-19

CYLINDER OF CROSS-SECTIONAL AREA A AND SLANT HEIGHT l

4.35 Volume $= Ah = Al \sin\theta$

4.36 Lateral surface area $= pl = \dfrac{ph}{\sin\theta} = ph \csc\theta$

Note that formulas 4.31 to 4.34 are special cases.

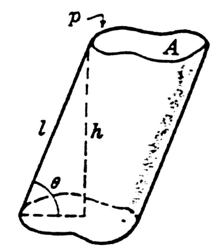


Fig. 4-20

RIGHT CIRCULAR CONE OF RADIUS r AND HEIGHT h

4.37 Volume $= \frac{1}{3}\pi r^2 h$

4.38 Lateral surface area $= \pi r\sqrt{r^2 + h^2} = \pi r l$

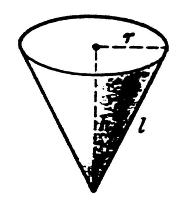


Fig. 4-21

PYRAMID OF BASE AREA A AND HEIGHT h

4.39 Volume $= \frac{1}{3}Ah$

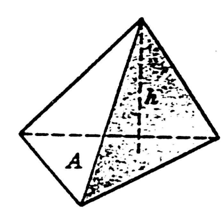


Fig. 4-22

SPHERICAL CAP OF RADIUS r AND HEIGHT h

4.40 Volume (shaded in figure) $= \frac{1}{3}\pi h^2(3r - h)$

4.41 Surface area $= 2\pi r h$

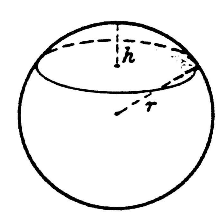


Fig. 4-23

FRUSTRUM OF RIGHT CIRCULAR CONE OF RADII a, b AND HEIGHT h

4.42 Volume $= \frac{1}{3}\pi h(a^2 + ab + b^2)$

4.43 Lateral surface area $= \pi(a + b)\sqrt{h^2 + (b - a)^2}$
$= \pi(a + b)l$

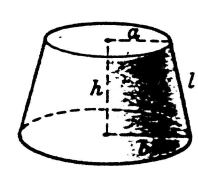


Fig. 4-24

SPHERICAL TRIANGLE OF ANGLES A, B, C ON SPHERE OF RADIUS r

4.44 Area of triangle $ABC = (A + B + C - \pi)r^2$

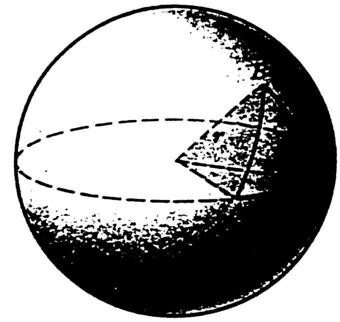


Fig. 4-25

TORUS OF INNER RADIUS a AND OUTER RADIUS b

4.45 Volume $= \frac{1}{4}\pi^2(a + b)(b - a)^2$

4.46 Surface area $= \pi^2(b^2 - a^2)$

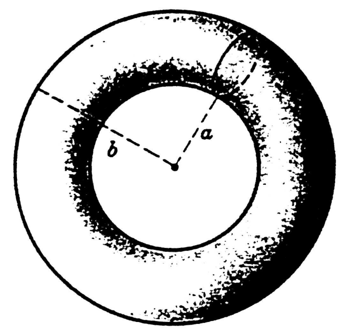


Fig. 4-26

ELLIPSOID OF SEMI-AXES a, b, c

4.47 Volume $= \frac{4}{3}\pi abc$

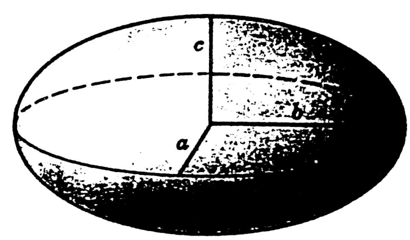


Fig. 4-27

PARABOLOID OF REVOLUTION

4.48 Volume $= \frac{1}{2}\pi b^2 a$

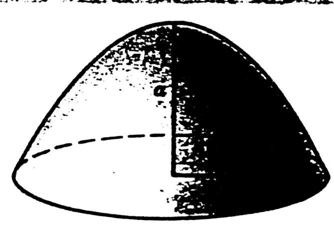


Fig. 4-28

5 TRIGONOMETRIC FUNCTIONS

DEFINITION OF TRIGONOMETRIC FUNCTIONS FOR A RIGHT TRIANGLE

Triangle ABC has a right angle (90°) at C and sides of length a, b, c. The trigonometric functions of angle A are defined as follows.

5.1 $\quad$ *sine* of A $= \sin A = \dfrac{a}{c} = \dfrac{\text{opposite}}{\text{hypotenuse}}$

5.2 $\quad$ *cosine* of A $= \cos A = \dfrac{b}{c} = \dfrac{\text{adjacent}}{\text{hypotenuse}}$

5.3 $\quad$ *tangent* of A $= \tan A = \dfrac{a}{b} = \dfrac{\text{opposite}}{\text{adjacent}}$

5.4 $\quad$ *cotangent* of A $= \cot A = \dfrac{b}{a} = \dfrac{\text{adjacent}}{\text{opposite}}$

5.5 $\quad$ *secant* of A $= \sec A = \dfrac{c}{b} = \dfrac{\text{hypotenuse}}{\text{adjacent}}$

5.6 $\quad$ *cosecant* of A $= \csc A = \dfrac{c}{a} = \dfrac{\text{hypotenuse}}{\text{opposite}}$

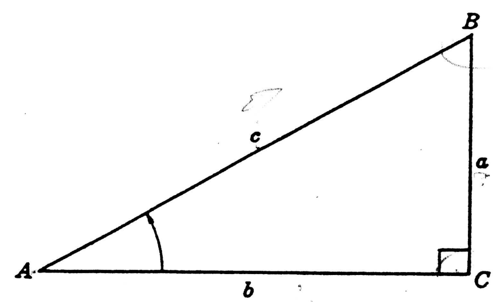


Fig. 5-1

EXTENSIONS TO ANGLES WHICH MAY BE GREATER THAN 90°

Consider an xy coordinate system [see Fig. 5-2 and 5-3 below]. A point P in the xy plane has coordinates (x, y) where x is considered as positive along OX and negative along OX' while y is positive along OY and negative along OY'. The distance from origin O to point P is positive and denoted by $r = \sqrt{x^2 + y^2}$. The angle A described *counterclockwise* from OX is considered *positive*. If it is described *clockwise* from OX it is considered *negative*. We call $X'OX$ and $Y'OY$ the x and y axis respectively.

The various quadrants are denoted by I, II, III and IV called the first, second, third and fourth quadrants respectively. In Fig. 5-2, for example, angle A is in the second quadrant while in Fig. 5-3 angle A is in the third quadrant.

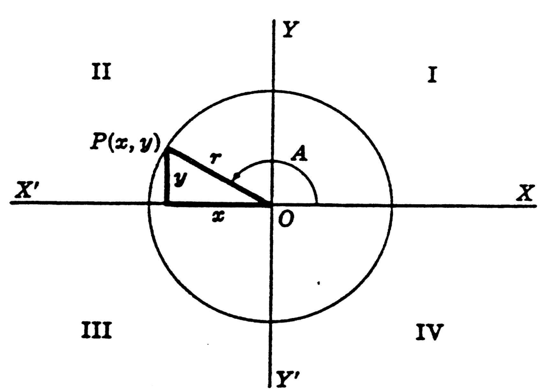


Fig. 5-2

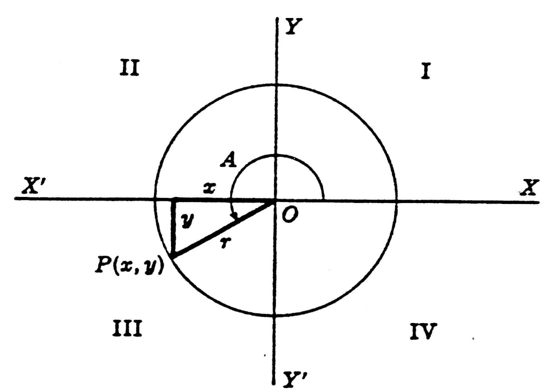


Fig. 5-3

BASIC MATH REVIEW FOR THE ENGINEERING TECHNICIAN EXAM

TRIGONOMETRY

Degrees and Radians

One circle is divided into 360 degrees or 2π radians

Conversions between degrees and radians are:

multiply ↓	by	to obtain
radians	$180/\pi$	degrees
degrees	$\frac{\pi}{180}$	radians

Angles

ACUTE less than 90°

obtuse more than 90°

reflex more than 180°

related angle differs from another by some multiple of 90°

Right angle equal to 90°

Complementary angles are two angles whose sum = 90°

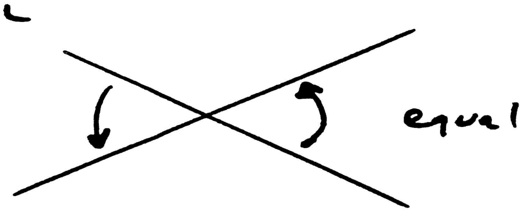
Supplementary angles are two angles whose sum = 180°

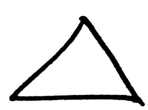
Basic Math Review for the Engineering Technician Exam

Trigonometry

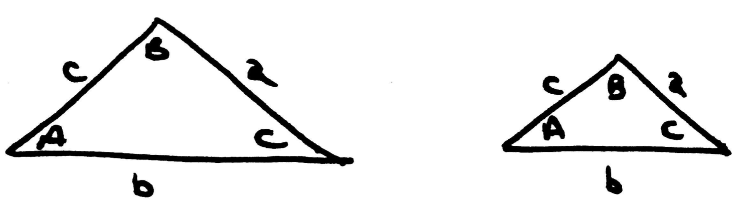
Adjacent Angles are known to share a common vertex and one interior side.

Vertical Angles have a common vertex and are equal



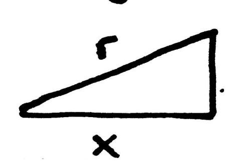
Triangles:  Sum of Angles Add up to 180°

In Similar Angles corresponding Angles are equal and corresponding sides are in proportion.



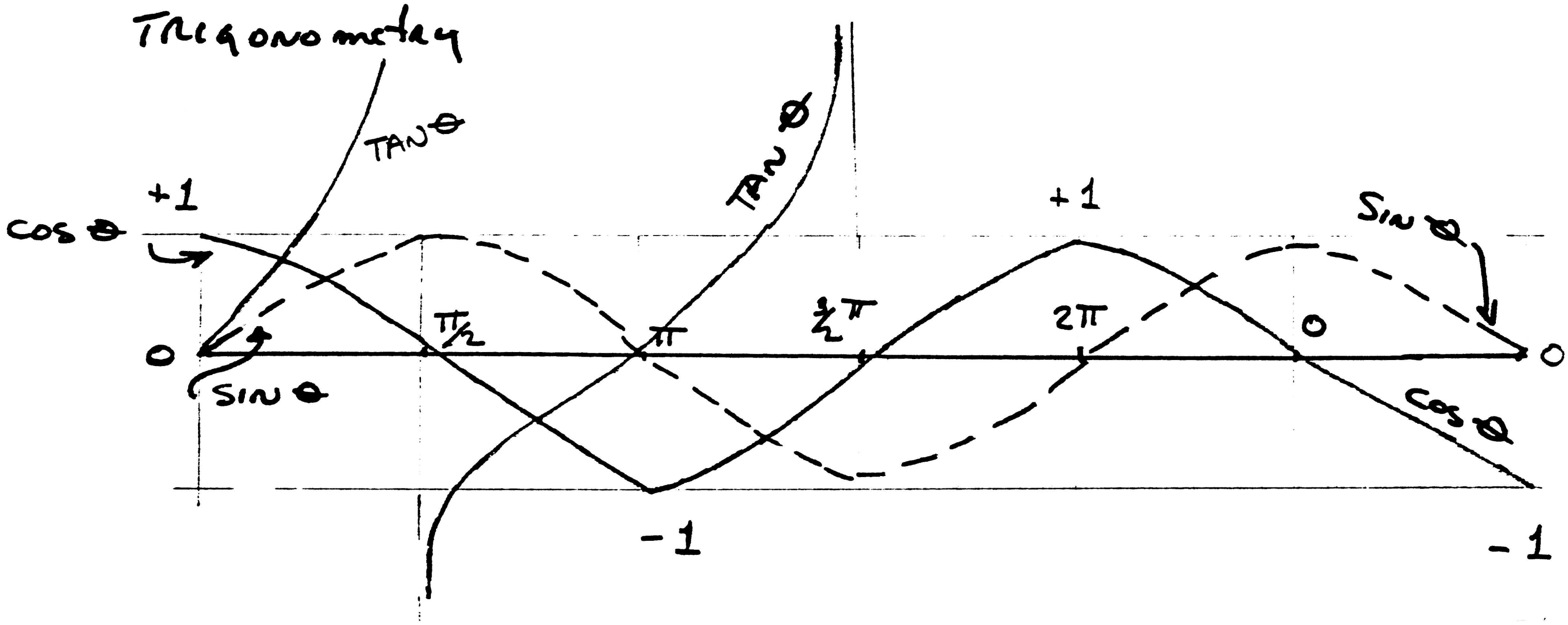
Right Triangle is a Triangle in which one of the Angles is 90°. The remaining two Angles are complementary.

Pythagorean Theorem $x^2+y^2=r^2$

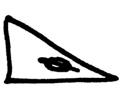


Basic Math Review for the Engineering Technician Exam

Trigonometry



Graphs of Sine, Cosine Tangent Functions



Functions of Related Angles

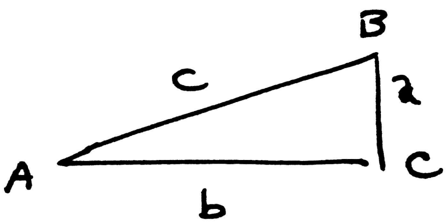
	θ	−θ	90° − θ	90° + θ	180° − θ	180° + θ
SIN		− sin θ	cos θ	cos θ	sin θ	− sen θ
COS		cos θ	sen θ	− sen θ	− cos θ	− cos θ
TAN		− TAN θ	cot θ	− cot θ	− TAN θ	TAN θ

Trigonometric Identities

$$\text{sen}^2\theta + \cos^2\theta = 1$$

Basic Math for the Engineering Technician Exam

General Triangles



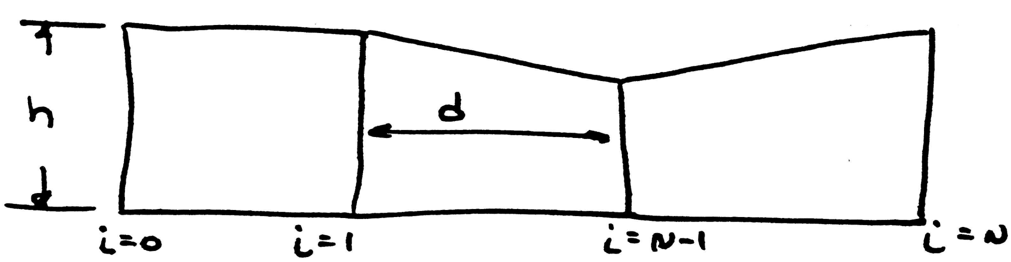
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\sin A = \frac{2}{bc}\sqrt{s(s-a)(s-b)(s-c)} \qquad s = \frac{1}{2}(a+b+c)$$

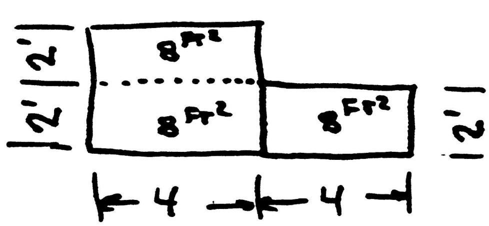
Analytic Geometry

Areas Trapezoidal Rule



$$A = \frac{d}{2}\left[h_0 + h_N + 2\sum_{i=1}^{N-1} h_i\right]$$

Example:



Total Area = 24 ft²

$d = 4'$
$h_0 = 4'$
$h_N = 2'$
$h_i = 4'$

By Rule

$$A = \frac{4}{2}\left[4' + 2' + 2\left[\sum_1^1 4\right]\right]$$

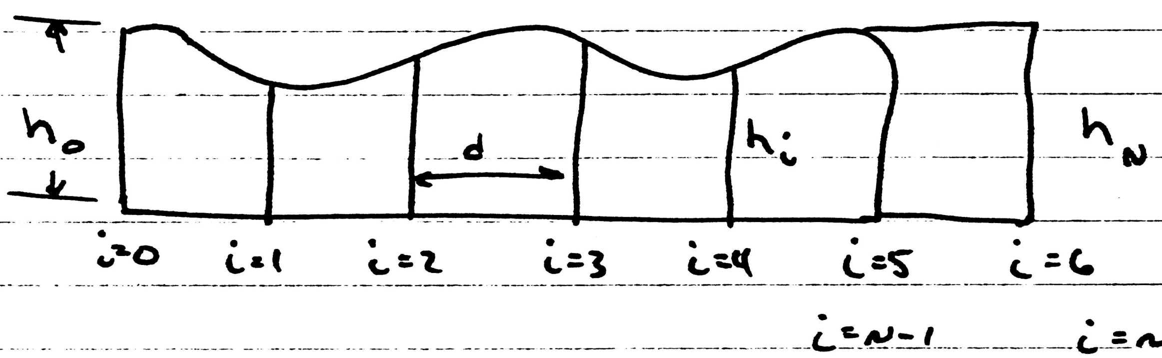
$$A = 2[14] = 24\,Ft^2$$

Basic Math For the Engineering Technician Exam

Analytic Geometry

Simpson's Rule

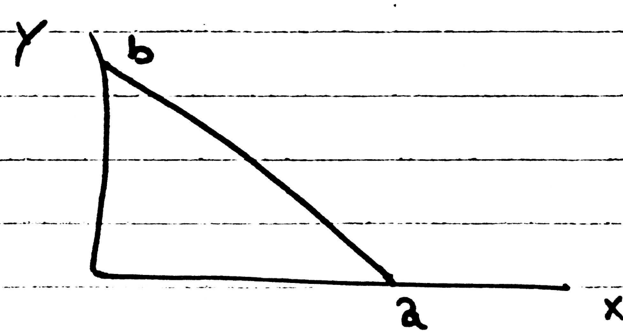
If the irregular side of each cell is curved use Simpson's rule for Area



N must be <u>even</u>

$$A = \frac{d}{3}\left[h_0 + h_N + 4\sum_{\substack{i \text{ odd} \\ i=1}}^{N-1} h_i + 2\sum_{\substack{i \text{ even} \\ i=2}}^{N-2} h_i\right]$$

Straight Lines



Slope of line = m

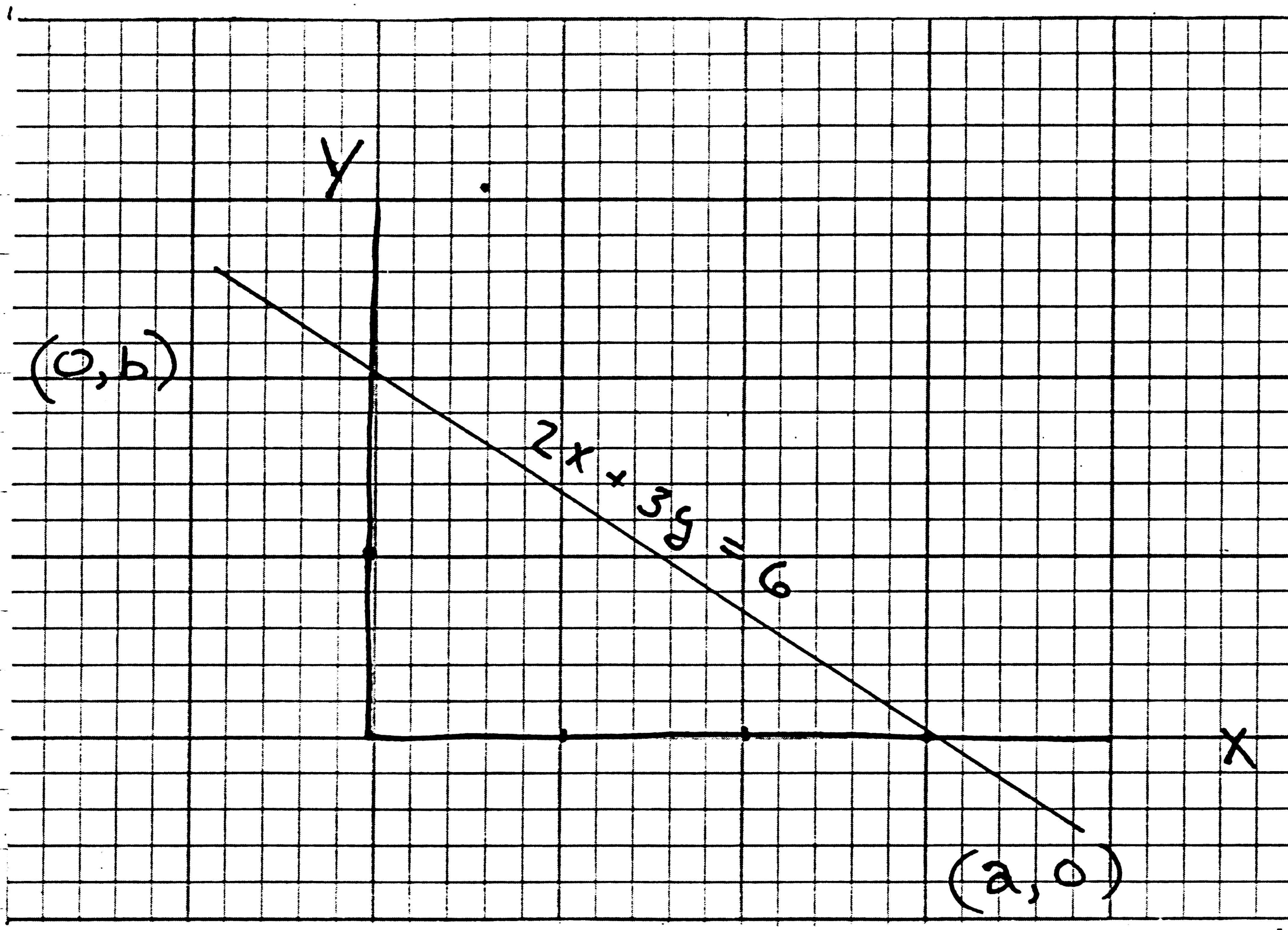
Y intercept = b

X intercept = a

Basic Math for the Engineering Technician Exam

Analytic Geometry

Straight Lines



Equation of a Line

$y = mx + b$ slope of line $= m$ $b = y$ intercept

$$m = \frac{\Delta y}{\Delta x}$$

Find slope of line whose equation is $2x + 3y = 6$

1) Transform equation into form $y = mx + b$

BASIC MATH FOR THE ENGINEERING TECHNICIAN EXAM

Analytic Geometry

Straight Lines

$$2x + 3y = 6$$

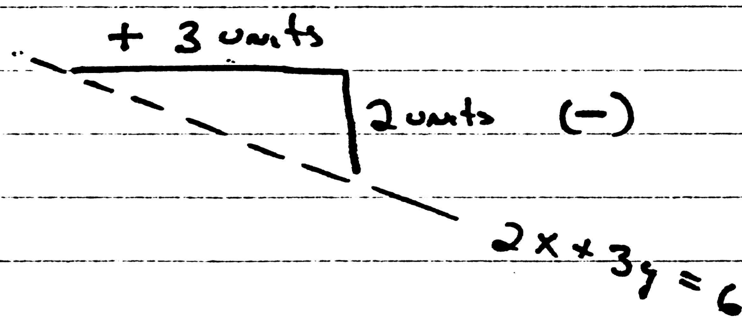
$$3y = 6 - 2x$$

$$y = \frac{6}{3} - \frac{2}{3}x$$

$$y = -\tfrac{2}{3}x + 3$$

Slope of Line $= -\tfrac{2}{3} = m$

This means that for every 3 units that X is y drops 2 units



Intersection of Two Lines

$$A_1 x_1 + B_1 y_1 + C_1 = 0$$

$$A_2 x_2 + B_2 y_2 + C_2 = 0$$

$$x = \frac{B_2 C_1 - B_1 C_2}{A_2 B_1 - A_1 B_2}$$

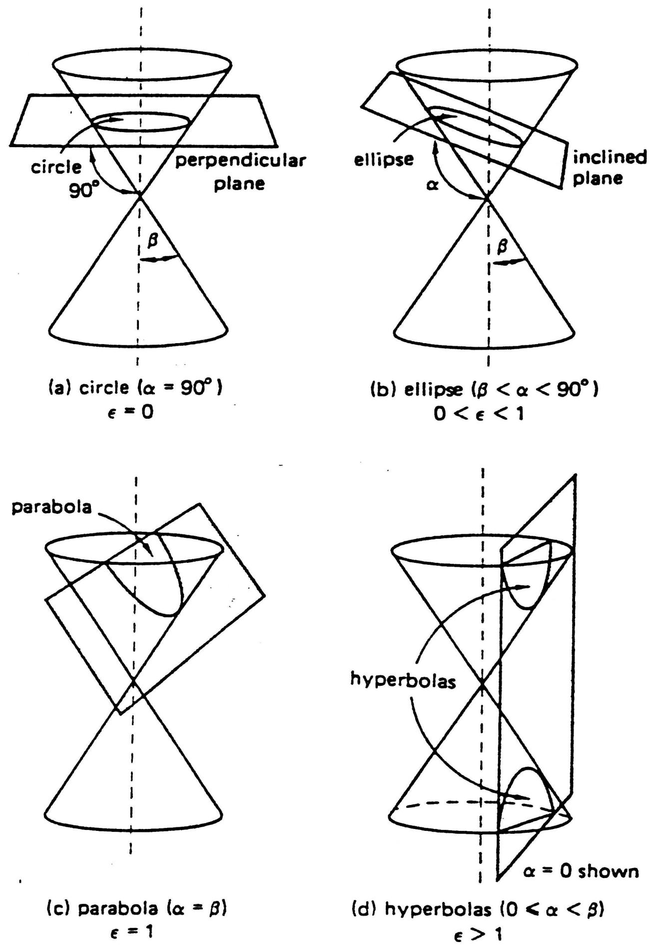
$$y = \frac{A_1 C_2 - A_2 C_1}{A_2 B_1 - A_1 B_2}$$

Basic Math For The Engineering Technician Exam

Analytic Geometry

Conic Sections:

1



(a) circle ($\alpha = 90°$)
$\epsilon = 0$

(b) ellipse ($\beta < \alpha < 90°$)
$0 < \epsilon < 1$

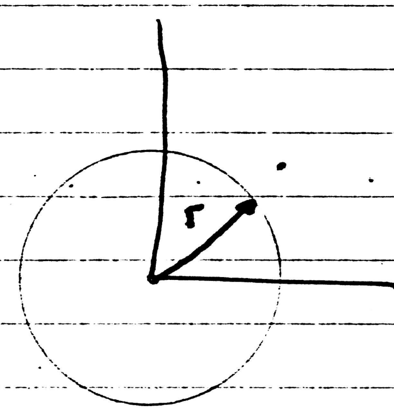
(c) parabola ($\alpha = \beta$)
$\epsilon = 1$

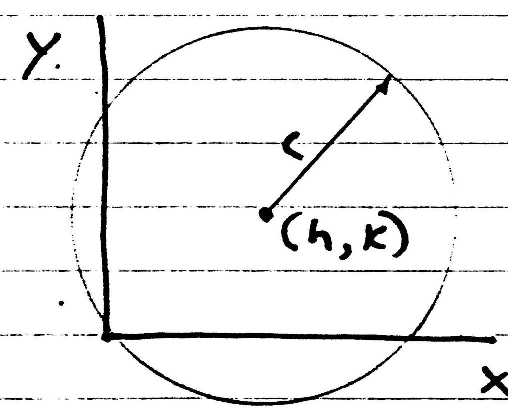
(d) hyperbolas ($0 < \alpha < \beta$)
$\epsilon > 1$

Circle:

$$Ax^2 + Ay^2 + Dx + Ey + F = 0$$

$$x^2 + y^2 = r^2$$





$$(x-h)^2 + (y-k)^2 = r^2$$

$$h = \frac{-D}{2A} \qquad k = \frac{-E}{2A}$$

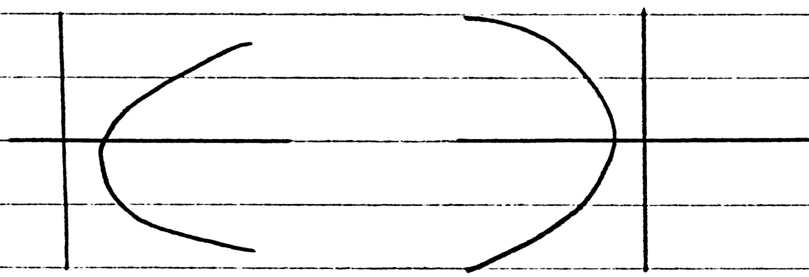
BASIC MATH FOR The Engineering TECHNICIAN EXAM

Analytic Geometry

PARABOLA:

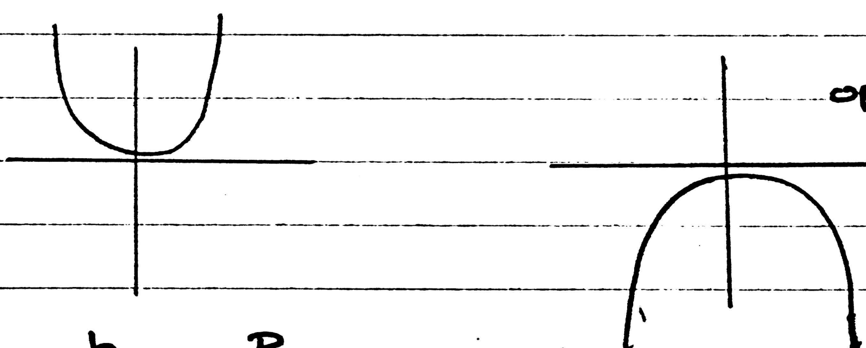
There are 2 types of parabola - those that open right and left and those that open up and down

Horizontal $(y-k)^2 = 4p(x-h)$



$y^2 = 4px$

vertex at origin

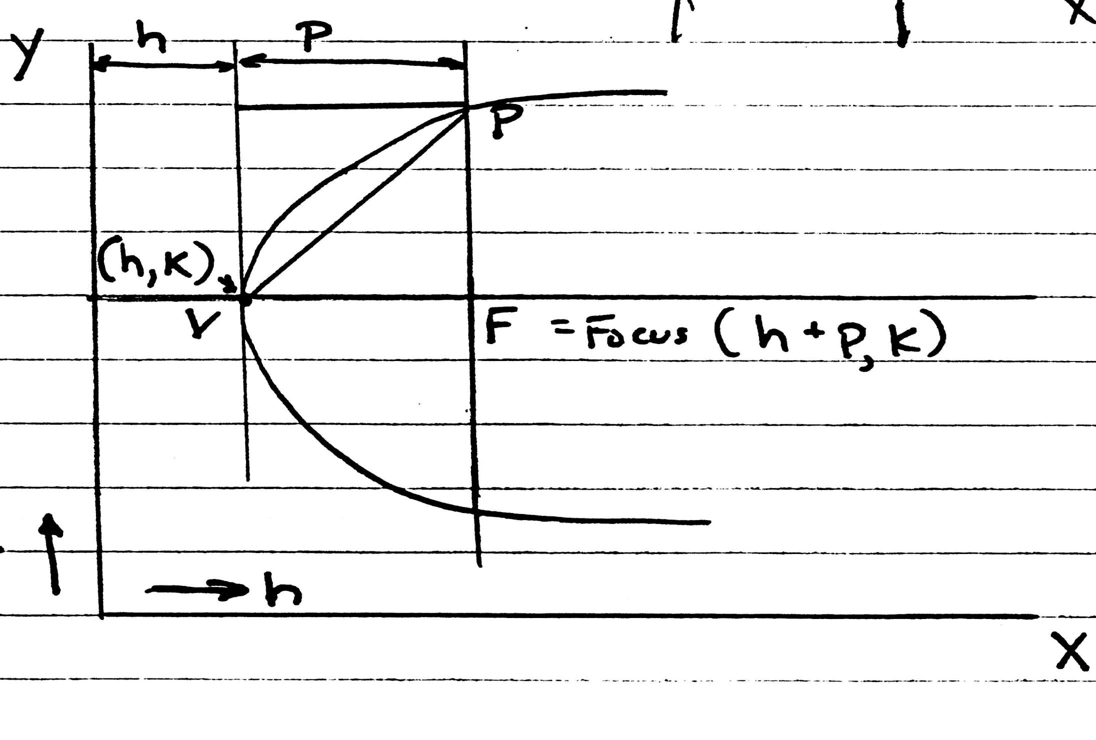


opens vertically

$(x-h)^2 = 4p(y-k)$

$x^2 = 4py$

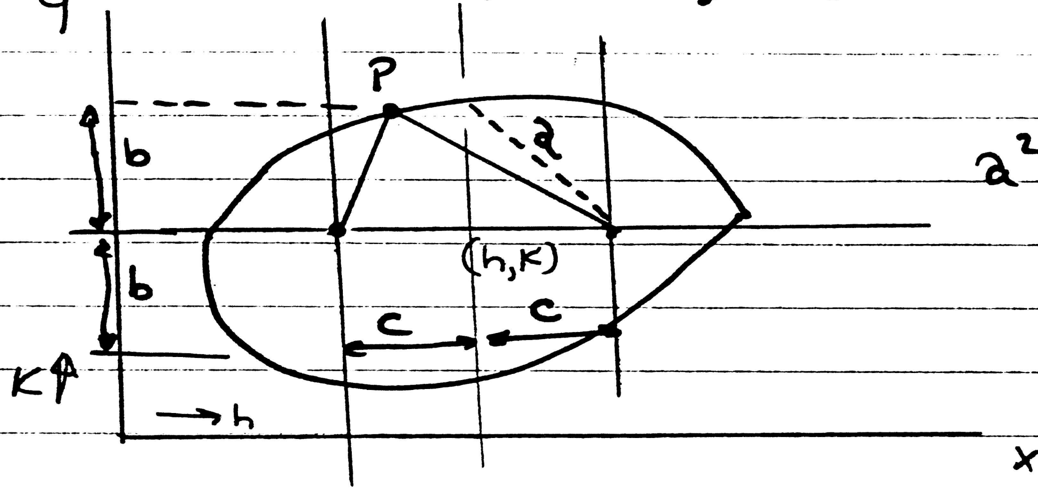
vertex at origin



Analytic Geometry

ELLIPSE

$$F_1P + PF_2 = 2a$$



$$a^2 = b^2 + c^2$$

$$Ax^2 + Cy^2 + Dx + Ey + F = 0$$

$AC > 0$

$A \neq C$

Aspect ratio of Ellipse $= \frac{a}{b}$

eccentricity of ellipse $= \epsilon = \frac{\sqrt{a^2 - b^2}}{a} < 1$

Always Less Than 1

IF eccentricity is zero

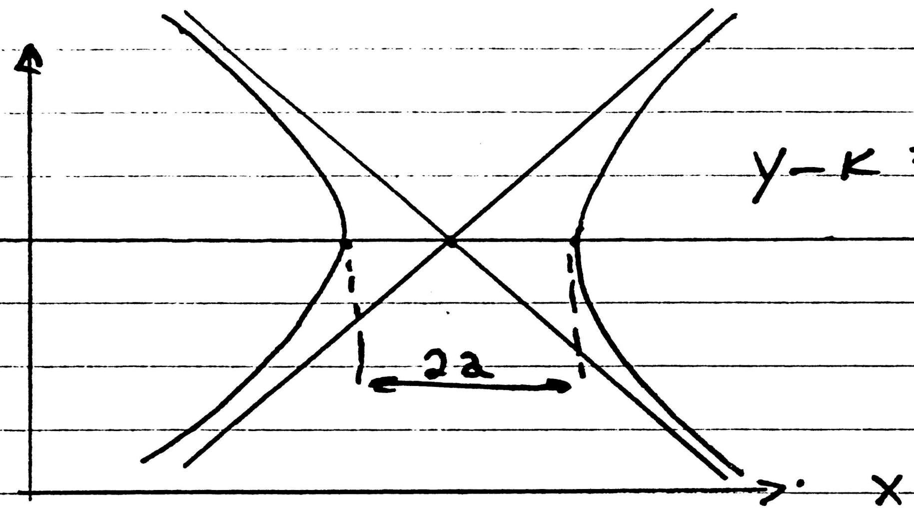
The figure is A circle.

BASIC MATH FOR THE ENGINEERING TECHNICIAN EXAM

ANALYTIC GEOMETRY

HYPERBOLA

y



$$y - k = \frac{b}{a}(x - h)$$

IF THE ASYMPTOTES ARE THE X AND Y AXIS THE EQUATION OF THE HYPERBOLE IS SIMPLY

$$xy = \pm \frac{a^2}{2}$$

SPHERE

3 TERMS

$$(x-h)^2 + (y-k)^2 + (z-l)^2 = r^2$$

$$Ax^2 + Ay^2 + Az^2 + Bx + Cy + Dz + E = 0$$

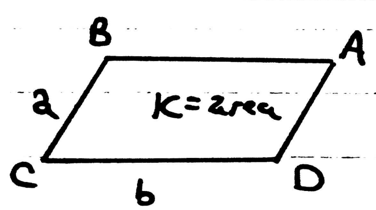
radius of sphere =

$$r = \sqrt{\frac{B^2 + C^2 + D^2}{4A^2} - \frac{E}{A}}$$

Methods of Solving Oblique Triangles

<table>
<tr><th>CASE
GIVEN</th><th>Methods of Solution</th></tr>
<tr><td>Side, Angle, Angle

Side, Side, Angle</td><td>Use Law of Sines
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$</td></tr>
<tr><td>Side, Angle, Side</td><td>Use the Law of Cosines
$$c^2 = a^2 + b^2 - 2ab\cos C$$</td></tr>
<tr><td>Side, Side, Side</td><td>Use the Law of Cosines to find the Angles
$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$</td></tr>
<tr><td>Area of A Parallelogram

$K = ab\sin C$</td><td>
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