

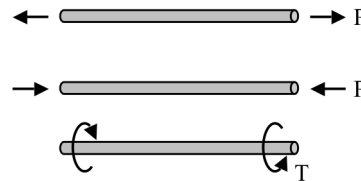
Shear Force and Bending Moment in Beams

Beam Loads

The way a part is loaded determines whether it is called a tensile or compressive member, a torsional shaft, or a beam. If you take a 1/2 inch diameter steel rod and pull it lengthwise, the rod will develop a tensile stress $\sigma = P / A$ where A is the cross-sectional area of the rod.

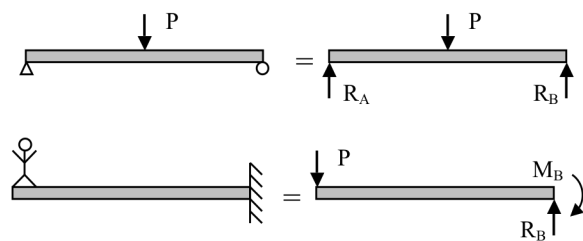
Loading the rod in tension parallel to its axis makes the rod a tensile member; loading it in compression parallel to its axis makes it a compressive member.

If you twist the steel rod with torque T, then we call it a torsional shaft.



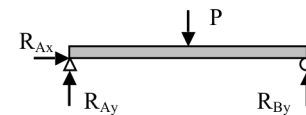
If loading is perpendicular (transverse) to its axis so that the rod bends, then the rod is called a beam. You can load a beam with point loads, uniformly distributed loads, or nonuniformly distributed loads.

A student standing on the end of a diving board is an example of a point load: a force applied at a single point on the beam. This point load P could be the weight of an object on the beam, or it could be a load applied by a cable or rod attached to the beam at that point.



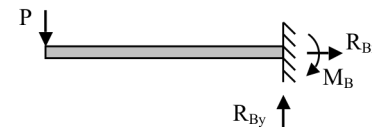
For each loaded beam we draw beam reactions: forces R_A and R_B for a simply supported beam; force R_B and moment M_B for a cantilever beam.

The symbols for supports indicate the kinds of reactions that develop at the support. For example, support "A" is pinned, like a hinge, so the symbol is a triangle. A pinned support may have vertical and horizontal reaction forces, but the beam at the right has no applied horizontal loads, therefore $R_{Ax} = 0$. In the beam problems that follow, there are no applied horizontal forces, so the horizontal reaction force is zero, and the vertical reaction forces R_{Ay} and R_{By} are abbreviated R_A and R_B .



If the beam sits on a roller that allows the beam to move freely horizontally, then the symbol is a circle. A roller support has only a vertical reaction force.

A cantilever beam is embedded in a wall, therefore the beam has vertical and horizontal reaction forces as well as a reaction moment. The horizontal reaction force R_{Bx} is zero as long as there are no horizontal applied forces, so the vertical reaction force R_{By} is usually abbreviated R_B .



The weight of a beam is an example of a uniform distributed load. The weight per unit length, w , typically has units of lb./ft., kips/ft., or kN/m. Consider a wide-flange beam, or “W-beam,” having a cross-section that looks like a Courier font capital letter I. The U.S. Customary W-beam designation system has two numbers: the first is the nominal depth, and the second is the weight per unit length. For example, a W24×162 beam has a nominal depth of 24 inches and a weight per unit length of 162 lb./ft. If the beam is 10 feet long, then the total weight, W , of the beam is

$$W = wL = \frac{162 \text{ lb.} \cdot 10 \text{ ft.}}{\text{ft.}} = 1620 \text{ lb.}$$

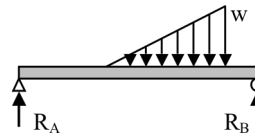
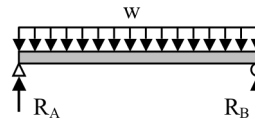
Canada uses SI (metric) units to specify W-beams. These beams are designated by mass, not weight: a W250×115 wide flange beam has a nominal depth of 250 mm and a **mass** per unit length of 115 kg/m. From Newton’s Second Law, force = mass × acceleration, or weight = mass × the acceleration of gravity. The SI unit of force is the newton (N), defined as $N = \text{kg m/s}^2$, and the acceleration of gravity is 9.81 m/s^2 . The weight per unit length of a

$$W250 \times 115 \text{ beam is } w = \frac{115 \text{ kg}}{\text{m}} \frac{9.81 \text{ m}}{\text{s}^2} \frac{\text{N}}{\text{kg m}} \frac{\text{kN}}{10^3 \text{ N}} = 1.13 \text{ kN/m}.$$

If the beam is 4 m long, then the total weight, W , of the beam is

$$W = wL = \frac{1.13 \text{ kN} \cdot 4 \text{ m}}{\text{m}} = 4.51 \text{ kN}.$$

A distributed load may run the length of the beam, may run along a portion of the beam, or may be nonuniform, like this tapered distributed load.



Reactions for Simply-Supported Simple Beams

You can calculate the reaction forces for a symmetrically-loaded beam by dividing the total load by 2, because each end of the beam carries half the load.

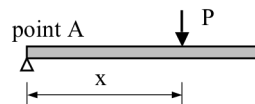
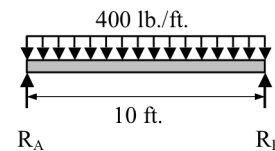
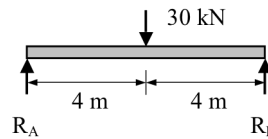
The reactions for the beam with a point load are $R_A = R_B = P \div 2$.

In this example, $R_A = R_B = 30 \text{ kN} \div 2 = 15 \text{ kN}$.

The reaction forces for a beam with a uniform distributed load are $R_A = R_B = wL \div 2$.

In this example, $R_A = R_B = \left(\frac{400 \text{ lb} \cdot 10 \text{ ft.}}{\text{ft.}} \right) \div 2 = 2000 \text{ lb.}$

If the loading is not symmetrical, then the beam reactions are calculated using Statics equations: the sum of the vertical forces equals zero, and the sum of the moments about a point equals zero. From Statics, you know how to calculate the moment about a point: in this example, $M_A = P x$.



In a beam problem, you can pick a point at either support. In this example, select point A. Since moment has a magnitude and a direction (clockwise or counterclockwise), we need to establish a convention for positive and negative moments. We'll select counterclockwise as positive, symbolized as \curvearrowright , and start adding up the moments.

The 40 kN load acts at a distance of 3m from point A. Think of point A as a hinge point...the 40 kN load causes the beam to rotate clockwise about point A, so the moment is negative.

The reaction force R_B acts at a distance of 10m from point A, and causes the beam to rotate counterclockwise about point A, so the moment is positive.

The moment about point A is $\curvearrowright \sum M_A = 0 = -40\text{kN} \cdot 3\text{m} + R_B \cdot 10\text{m}$. Now solve for the reaction force $R_B = \frac{40\text{kN} \cdot 3\text{m}}{10\text{m}} = 12\text{kN}$.

Use the sum of the forces in the vertical direction to calculate the other reaction force. Forces have magnitude and direction; pick upwards as positive, so $\sum F_y = 0 = R_A - 40\text{kN} + 12\text{kN}$.

Now solve for the reaction force $R_A = 40\text{kN} - 12\text{kN} = 28\text{kN}$.

You can check your answer by solving the sum of the moments about point B.

Use the same technique for beams with more than one point load.

The moment about point A is

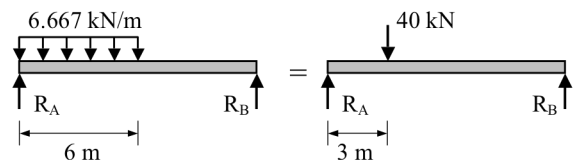
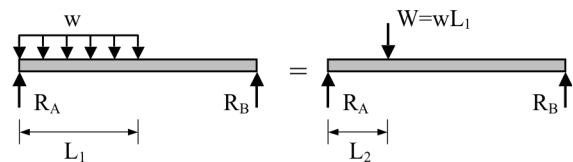
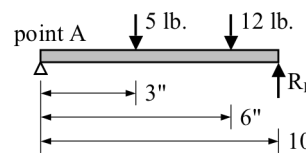
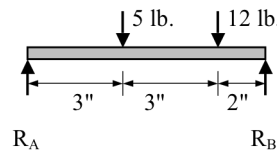
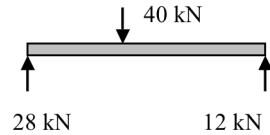
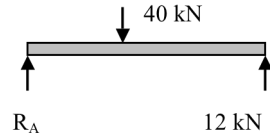
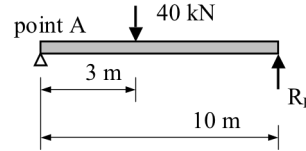
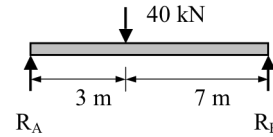
$$\curvearrowright \sum M_A = 0 = -5\text{lb} \cdot 3'' - 12\text{lb} \cdot 6'' + R_B \cdot 10''$$

Solve for the reaction force $R_B = \frac{5\text{lb} \cdot 3\text{in.} + 12\text{lb} \cdot 6\text{in.}}{10\text{in.}} = 8.7\text{lb}$.

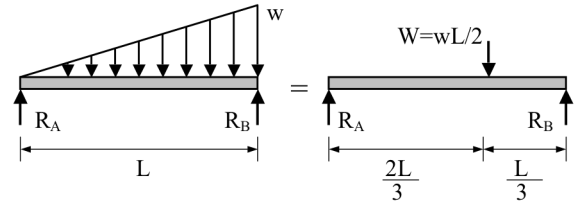
Sum of the forces $\sum F_y = 0 = R_A - 5\text{lb.} - 12\text{lb.} + 8.7\text{lb.}$

Solve for $R_A = 5\text{lb.} + 12\text{lb.} - 8.7\text{lb.} = 8.3\text{lb.}$

If a uniformly distributed load is not symmetrical, then we need to convert the distributed load into a point load equivalent to the total load $W = wL_1$ where L_1 is the length of the distributed load. The equivalent point load is located at the centroid of the distributed load...the center of the rectangle. Use the equivalent load diagram for calculating the reaction forces. For example, if $L_1 = 6\text{m}$, total length $L = 10\text{m}$, and $w = 6.667\text{kN/m}$, then the equivalent point load $W = wL_1 = \frac{6.667\text{kN}}{\text{m}} \cdot 6\text{m} = 40\text{kN}$, and the location of the equivalent point load $L_2 = 3\text{m}$ from the left end. The numerical solution is identical to the previous example.

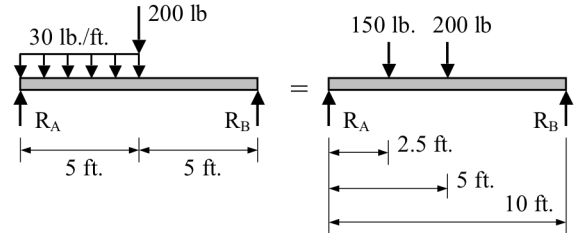


Use the same approach for a nonuniformly distributed load. Again, the location of the equivalent load is at the centroid of the distributed load. The centroid of a triangle is one-third the distance from the wide end of the triangle, so the location of the equivalent load is one-third of the length measured from the right end of this beam.



The load varies from 0 at the left end to w at the right end; therefore, the total load is the average of these loads times the beam length: $W = \frac{0 + w}{2} L = wL / 2$.

If the beam has a point load and a distributed load, draw an equivalent load diagram with the applied point load and the equivalent point load. The moment about point A is



$$\sum M_A = 0 = -150\text{lb.} \cdot 2.5\text{ft.} - 200\text{lb.} \cdot 5\text{ft.} + R_B \cdot 10\text{ft.}$$

Solve for the reaction force

$$R_B = \frac{150\text{lb.} \cdot 2.5\text{ft.} + 200\text{lb.} \cdot 5\text{ft.}}{10\text{ft.}} = 137.5\text{lb.}$$

$$\sum F_y = 0 = R_A - 150\text{lb.} - 200\text{lb.} + 137.5\text{lb.}$$

$$\text{Solve for } R_A = 150\text{lb.} + 200\text{lb.} - 137.5\text{lb.} = 212.5\text{lb.}$$

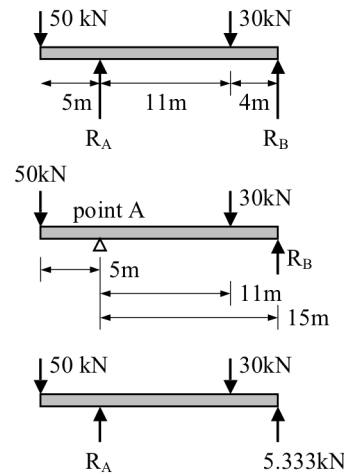
Overhanging and Cantilever Beams

A simply-supported beam is supported by pinned connections at both ends; all applied loads lie between these two points. An overhanging beam is loaded beyond the supports. If the beam extends beyond the supports, then loads applied beyond the supports are called cantilever loads. The solution method is the same as for simply-supported beams: use the sum of the moments about one of the support points to find the reaction at the other support point.

Here, the sum of the moments about point A is

$$\sum M_A = 0 = 50\text{kN} \cdot 5\text{m} - 30\text{kN} \cdot 11\text{m} + R_B \cdot 15\text{m}$$

Note the 50kN load produces a positive (counterclockwise) moment about point A.



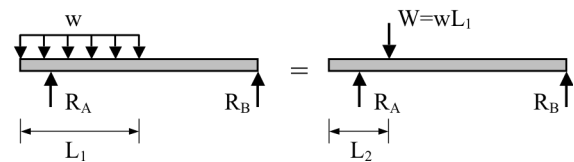
$$\text{Solving, we get } R_B = \frac{-50\text{kN} \cdot 5\text{m} + 30\text{kN} \cdot 11\text{m}}{15\text{m}} = 5.333\text{kN}.$$

Sum of the vertical forces

$$\sum F_y = 0 = R_A - 50\text{kN} - 30\text{kN} + 5.333\text{kN}.$$

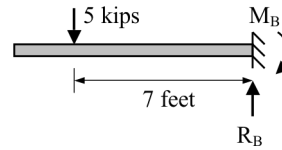
$$\text{Solve for } R_A = 50\text{kN} + 30\text{kN} - 5.333\text{kN} = 74.667\text{kN}.$$

Solve for the reactions to a cantilever distributed load the same way as for a simply-supported beam: draw an equivalent load diagram, then use the sum of the moments and the sum of the forces to find the reactions. Look at the diagram carefully: distance L_2 is the distance from W to the end of the beam; it is *not* the distance from W to R_A , which you will need for the sum of the moments calculation.



A cantilever beam with a single support has a reaction force and a reaction moment. The reaction force R_B equals the sum of the applied forces on the beam, so $R_B = 5 \text{ kips}$. The moment reaction equals the sum of the moments about point B, so

$$M_B = 5 \text{ kips} \cdot 7 \text{ ft.} = 35 \text{ ft.kips.}$$



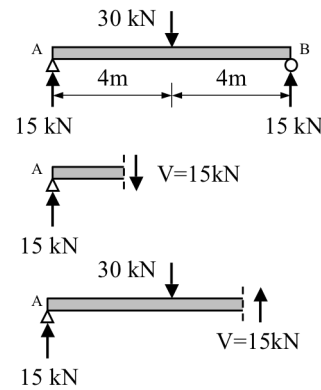
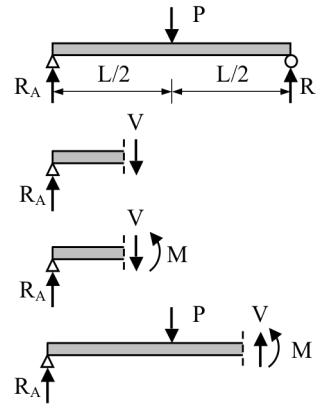
Shear Diagrams

Imagine a simply-supported beam with a point load at the mid-span. Cut the beam to the left of the point load, and draw a free-body diagram of the beam segment. In a free-body diagram, forces must balance. Therefore, a downward force at the cut edge balances the support reaction R_A . We call this shear force “V”.

The forces are in balance, but the segment wants to spin clockwise about point A. To counteract this tendency to spin, a torque, or moment, develops within the beam to prevent this rotation.

Cut the beam to the right of the point load, and draw the free-body diagram. Since P is larger than R_A , force V points upwards.

We can find the value of V using Sum of the Forces in the vertical direction. From point A to the applied load, $V = 15 \text{ kN}$ downward; from the applied load to point B, $V = 15 \text{ kN}$ upward.



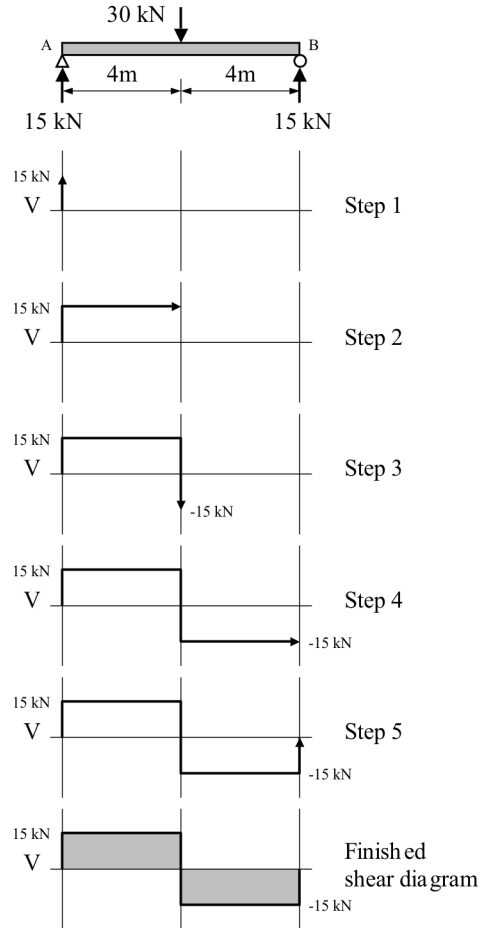
We can sketch V as a function of location along the beam using a Shear Diagram.

Draw vertical construction lines below the load diagram wherever the applied loads and reactions occur. Draw a horizontal construction line, indicating zero shear load.

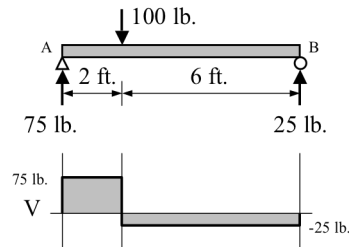
Next, draw the value of V along the length of the beam, as follows:

- Step 1: Starting at the left side of the shear diagram, go up 15kN, because R_A is 15kN upwards.
- Step 2: There are no additional loads on the beam until you get to the midspan, so the shear value remains at 15kN.
- Step 3: The applied load is 30 kN downwards, therefore the shear load is $15\text{kN} - 30\text{kN} = -15\text{kN}$.
- Step 4: There are no additional loads on the beam until you get to point B, so the shear value remains at -15kN.
- Step 5: At point B, the reaction force $R_B = 15\text{kN}$ upwards, therefore the shear load is $-15\text{kN} + 15\text{kN} = 0\text{kN}$. If you don't get to 0, you know you made a mistake someplace.

For readability, shear diagrams are often shaded.



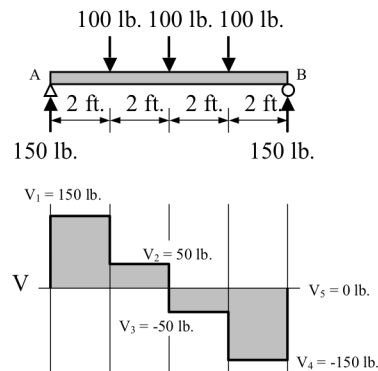
If the point load is not at the midspan, use Sum of the Moments and Sum of the Forces to calculate the reaction forces. Draw vertical construction lines below the applied loads and reaction forces, draw a horizontal line at zero shear, then draw the shear value along the length of the beam. Go up $R_A = 75\text{ lb.}$ at point A, go straight across to the applied load, down 100 lb. to -25 lb., straight across to point B, then up $R_B = 25\text{ lb.}$



Multiple point loads will give you a stepped shear diagram.

Calculate the values on the shear diagram as follows:

$$\begin{aligned}
 V_1 &= R_A = 150\text{ lb.} \\
 V_2 &= V_1 - 100\text{ lb.} = 50\text{ lb.} \\
 V_3 &= V_2 - 100\text{ lb.} = -50\text{ lb.} \\
 V_4 &= V_3 - 100\text{ lb.} = -150\text{ lb.} \\
 V_5 &= V_4 + R_B = -150\text{ lb.} + 150\text{ lb.} = 0\text{ lb.}
 \end{aligned}$$



A uniformly distributed load is like an infinite number of small point loads along the length of the beam, so the shear diagram is like a stepped multiple point load shear diagram with infinitely small steps. Since the loading is symmetrical, the reaction forces equal half the total load: $R_A = R_B = \left(\frac{3 \text{ kN} \cdot 4 \text{ m}}{m}\right) \div 2 = 6 \text{ kN}$.

Calculate the values on the shear diagram as follows:

$$V_1 = R_A = 6 \text{ kN}$$

$$V_2 = V_1 - \frac{3 \text{ kN} \cdot 4 \text{ m}}{m} = -6 \text{ kN}$$

$$V_3 = V_2 + R_B = -6 \text{ kN} + 6 \text{ kN} = 0 \text{ kN}$$

Some uniformly distributed loads do not extend along the entire length of a beam. Draw an equivalent load diagram to the right, and calculate the reaction forces. The equivalent point load

$$W = \frac{6 \text{ kN} \cdot 2 \text{ m}}{m} = 12 \text{ kN} . \text{ Sum of the moments about point A,}$$

$$\Sigma M_A = 0 = -12 \text{ kN} \cdot 3 \text{ m} + R_B \cdot 4 \text{ m} . \text{ Solving, we get}$$

$$R_B = \frac{12 \text{ kN} \cdot 3 \text{ m}}{4 \text{ m}} = 9 \text{ kN} . \text{ Sum of the forces gives us}$$

$$R_A = 12 \text{ kN} - 9 \text{ kN} = 3 \text{ kN} .$$

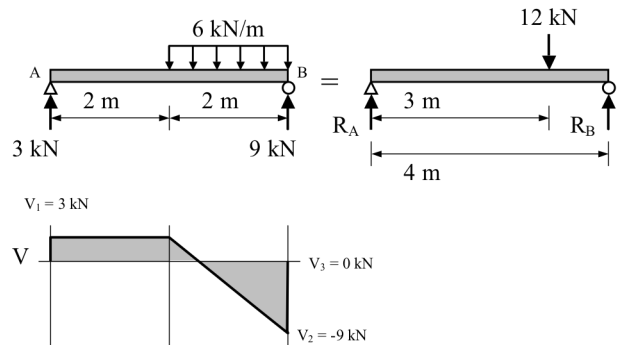
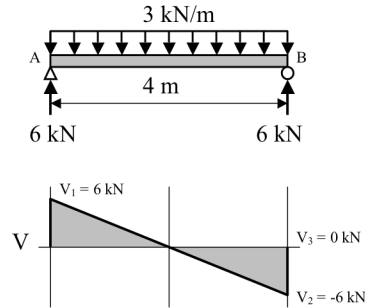
The shear diagram starts at point A with a positive 3 kN, goes straight across to the distributed load, drops diagonally to point B at a rate of 6 kN/m. Add R_B to reach $V = 0$.

Calculate the values on the shear diagram as follows:

$$V_1 = R_A = 3 \text{ kN}$$

$$V_2 = V_1 - \frac{6 \text{ kN} \cdot 2 \text{ m}}{m} = -9 \text{ kN}$$

$$V_3 = V_2 + R_B = -9 \text{ kN} + 9 \text{ kN} = 0 \text{ kN}$$



Solve for the reactions of a nonuniformly distributed load by drawing an equivalent load diagram to the right, with the equivalent point load $2/3$ of the distance from the sharp end of the triangle ($1/3$ of the distance from the blunt end). The equivalent point

load $W = \left(\frac{300 \text{ lb} \cdot 12 \text{ ft.}}{\text{ft.}} \right) \div 2 = 1800 \text{ lb.}$ Sum of the moments

about point A, $\Sigma M_A = 0 = -1800 \text{ lb.} \cdot 8 \text{ ft.} + R_B \cdot 12 \text{ ft.}$ Solving, we

get $R_B = \frac{1800 \text{ lb.} \cdot 8 \text{ ft.}}{12 \text{ ft.}} = 1200 \text{ lb.}$ Sum of the forces gives us

$$R_A = 1800 \text{ lb.} - 1200 \text{ lb.} = 600 \text{ lb.}$$

The shear diagram starts at point A with a positive 600 lb., then drops parabolically to point B. Calculate the values on the shear diagram as follows:

$$V_1 = R_A = 600 \text{ lb.}$$

$$V_2 = V_1 - \left(\frac{300 \text{ lb.} \cdot 12 \text{ ft.}}{\text{ft.}} \right) \div 2 = -1200 \text{ lb.}$$

$$V_3 = V_2 + R_B = -1200 \text{ lb.} + 1200 \text{ lb.} = 0 \text{ lb.}$$

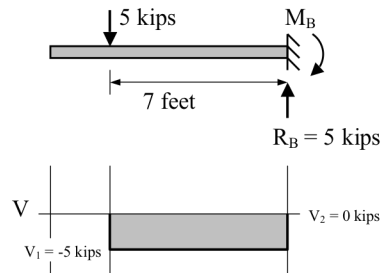
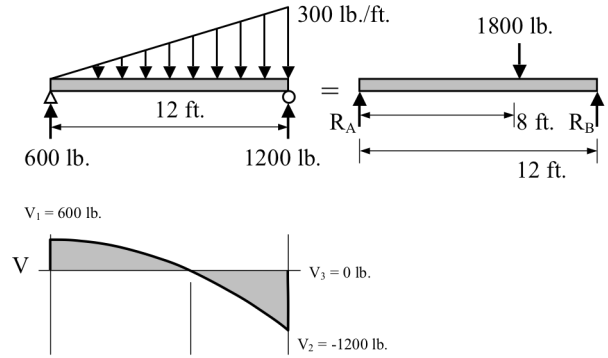
Look at the shear diagrams, and you can see that point loads create rectangles, uniform distributed loads create triangles, and nonuniformly distributed loads create parabolas.

Shear diagrams for cantilever beams follow the same rules as for simply supported beams. Since there is no reaction force at the left end, there is no shear load until we get to the applied load. Then, the shear load is negative (downward) until we get to the support, where R_B is positive (upward). Calculate the values on the shear diagram as follows:

$$V_1 = -5 \text{ kips}$$

$$V_2 = V_1 + R_B = -5 \text{ kips} + 5 \text{ kips} = 0 \text{ kips}$$

With a shear diagram, we can identify the location and size of the largest shear load in a beam. Therefore, we know the location of the largest shear stress, and we can calculate the value of this stress. Once we know the actual stress in the material, we can compare this values with the shear strength of the material, and we can know whether the beam will fail in shear. Shear diagrams are necessary for drawing bending moment diagrams (“moment diagrams”, for short), which we can use to identify the location and size of bending stresses that develop within beams. We can compare the actual bending stresses with the yield strength of the material, and we can know whether the beam will fail in bending.



Moment Diagrams

The moment about a point along a beam is defined as the distance from that point to a force acting perpendicular to the beam, so the units are force \times distance: ft. \cdot lb., in. \cdot lb., kip \cdot ft., N \cdot m, or kN \cdot m. We can graph the value of the bending moment along a beam by drawing a moment diagram.

To draw a moment diagram, sketch the value of the moment produced by the shear force V times the distance from the left end of the beam. At the first meter, $V = 15$ kN, so moment

$M_1 = 15 \text{ kN} \times 1 \text{ m} = 15 \text{ kN}\cdot\text{m}$
 At 2 m, $M_2 = M_1 + 15 \text{ kN} \times 1 \text{ m} = 30 \text{ kN}\cdot\text{m}$

At 3 m, $M_3 = M_2 + 15 \text{ kN} \times 1 \text{ m} = 45 \text{ kN}\cdot\text{m}$

At 4 m, $M_4 = M_3 + 15 \text{ kN} \times 1 \text{ m} = 60 \text{ kN}\cdot\text{m}$

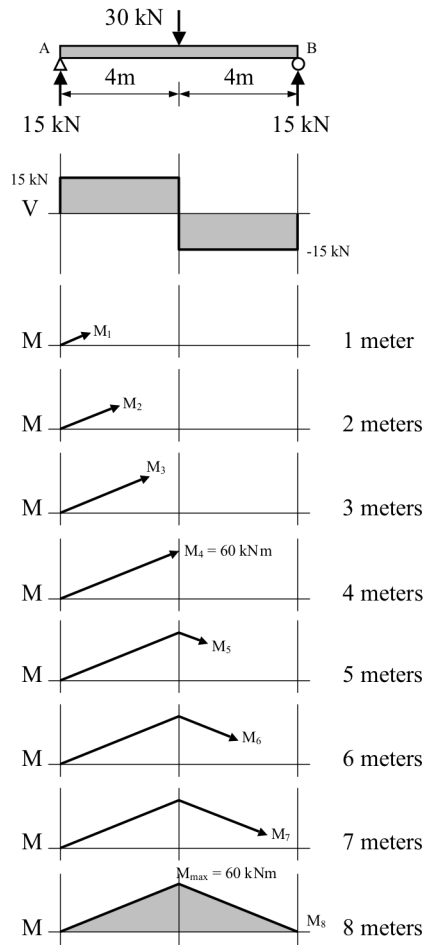
Beyond the midspan, V is negative, so at 5 m,
 $M_5 = M_4 - 15 \text{ kN} \times 1 \text{ m} = 45 \text{ kNm}$

At 6 m, $M_6 = M_5 - 15 \text{ kN} \times 1 \text{ m} = 30 \text{ kN}\cdot\text{m}$

At 7 m, $M_7 = M_6 - 15 \text{ kN} \times 1 \text{ m} = 15 \text{ kN}\cdot\text{m}$

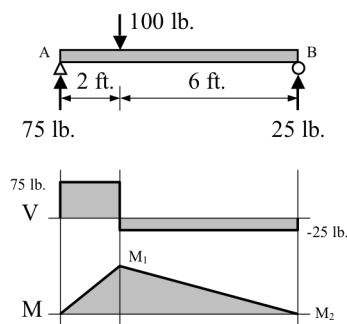
At 8 m, $M_8 = M_7 - 15 \text{ kN} \times 1 \text{ m} = 0 \text{ kN}\cdot\text{m}$

The value of the moment diagram at any point equals the area of the shear diagram up to that point. You can draw the moment diagram faster by calculating the area of the first rectangle in the shear diagram: $M_{\text{max}} = 15 \text{ kN} \times 4 \text{ m} = 60 \text{ kN}\cdot\text{m}$. Verify that $M_8 = 0$ by subtracting the area of the second rectangle in the shear diagram: $M_8 = M_{\text{max}} - 15 \text{ kN} \times 4 \text{ m} = 0 \text{ kN}\cdot\text{m}$



If the point load is not at the midspan, then the maximum moment will also be offset. The maximum moment is the area of the shear diagram up to the point load, $M_{\text{max}} = M_1 = 75 \text{ lb.} \times 2 \text{ ft.} = 150 \text{ ft.}\cdot\text{lb.}$ Check the moment at point B: $M_2 = M_1 - 25 \text{ lb.} \times 6 \text{ ft.} = 0 \text{ ft.}\cdot\text{lb.}$

If your M_2 in a problem like this is not zero, then you know there is an error someplace. Most likely, the mistake is in the reaction forces. You can draw a shear diagram that works with the wrong reaction forces, but you cannot draw a good moment diagram if R_A and R_B are wrong.



Multiple point loads will give you multiple rectangles on the shear diagram, and multiple triangles on the moment diagram.

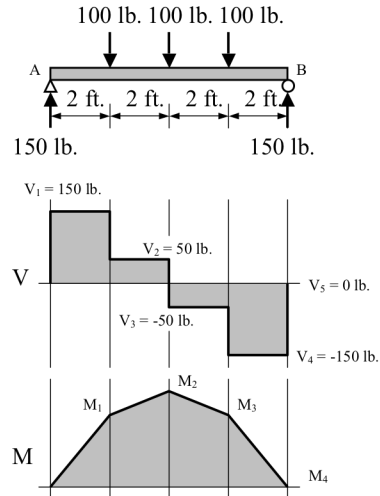
Calculate the values on the moment diagram as follows:

$$M_1 = 2 \text{ ft.} \times 150 \text{ lb.} = 300 \text{ ft.}\cdot\text{lb.}$$

$$M_2 = M_1 + 2 \text{ ft.} \times 50 \text{ lb.} = 400 \text{ ft.}\cdot\text{lb.}$$

$$M_3 = M_2 - 2 \text{ ft.} \times 50 \text{ lb.} = 300 \text{ ft.}\cdot\text{lb.}$$

$$M_4 = M_3 - 2 \text{ ft.} \times 150 \text{ lb.} = 0 \text{ ft.}\cdot\text{lb.}$$

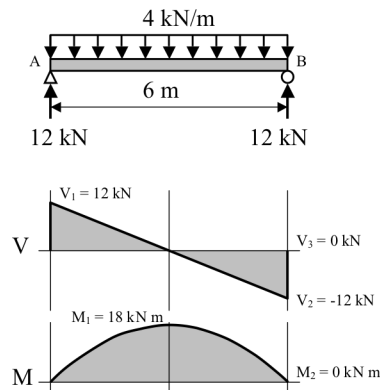


A uniformly distributed load produces a parabolic moment diagram. Close to point A, a large shear produces a steep slope in the moment diagram. As you approach the midspan, the smaller shear produces a shallower slope in the moment diagram. Beyond the midspan, an increasingly negative shear produces an increasingly steeper slope downwards.

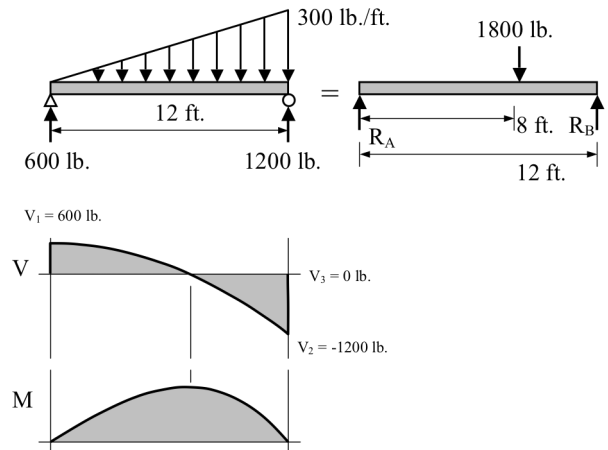
The maximum moment equals the area of the left-hand triangle. Subtract the area of the right-hand triangle to get the moment at point B.

$$M_1 = \frac{12 \text{ kN} \times 3 \text{ m}}{2} = 18 \text{ kN}\cdot\text{m}$$

$$M_2 = M_1 - \frac{12 \text{ kN} \times 3 \text{ m}}{2} = 0 \text{ kN}\cdot\text{m}$$

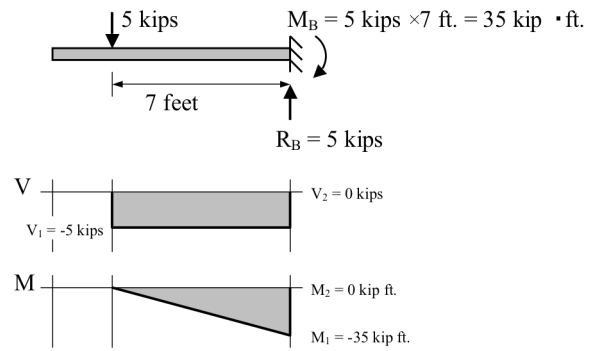


We've seen that a nonuniformly distributed load produces a parabolic shear diagram. The moment diagram looks like a parabola skewed to the right, with the maximum moment at the point where the shear diagram crosses the zero-load axis.



The shear diagram of this cantilever beam is negative, so the moment diagram decreases from the applied load to the support. The area of the shear diagram gives us the value of $M_1 = -5 \text{ kips} \times 7 \text{ ft.} = -35 \text{ kip}\cdot\text{ft.}$

$$M_2 = -35 \text{ kip}\cdot\text{ft.} + M_B = 0 \text{ kip}\cdot\text{ft.}$$



The shear diagram of a cantilever beam with a uniform distributed load is a triangle. Use an equivalent load diagram to find the reaction force and reaction moment.

$$W = wL = \frac{2 \text{ kN}}{\text{m}} \frac{3 \text{ m}}{\text{m}} = 6 \text{ kN}$$

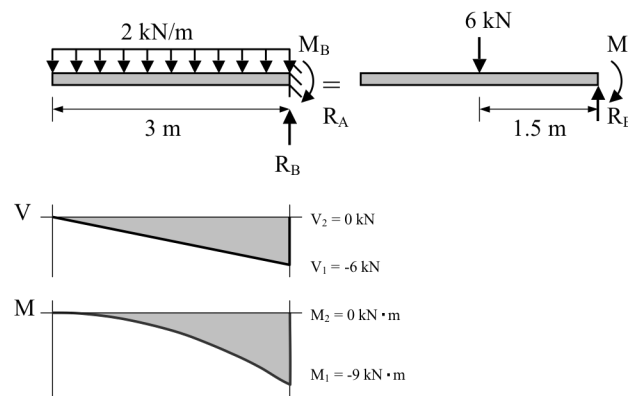
$$R_B = W = 6 \text{ kN}$$

$$M_B = 6 \text{ kN} \times 1.5 \text{ m} = 9 \text{ kN}\cdot\text{m}$$

The area of the shear diagram gives us the value of

$$M_1 = -6 \text{ kN} \times 3 \text{ m} = -9 \text{ kN}\cdot\text{m}$$

$$M_2 = -9 \text{ kN}\cdot\text{m} + M_B = 0 \text{ kN}\cdot\text{m}$$



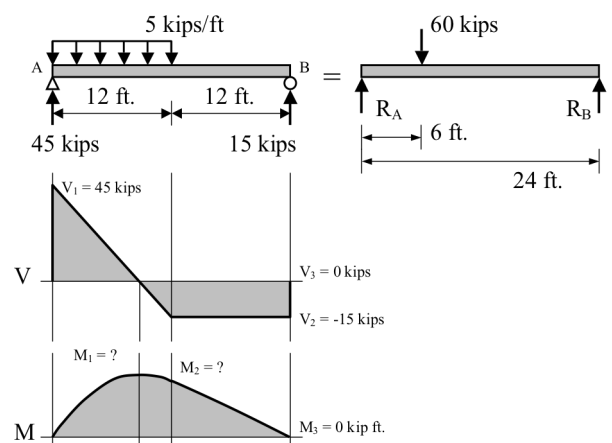
With this beam, a uniformly distributed load extends along half of the beam length. Use an equivalent load diagram to determine the reaction forces.

The equivalent point load $W = \frac{5 \text{ kips}}{\text{ft.}} \frac{12 \text{ ft.}}{\text{ft.}} = 60 \text{ kips}$. Sum of the moments about point A, $\Sigma M_A = 0 = -60 \text{ kips} \cdot 6 \text{ ft.} + R_B \cdot 24 \text{ ft.}$. Solving, we get $R_B = \frac{60 \text{ kips} \cdot 6 \text{ ft.}}{24 \text{ ft.}} = 15 \text{ kips}$. Sum of the forces gives us

$$R_A = 60 \text{ kips} - 15 \text{ kips} = 45 \text{ kips}$$

The shear curve starts at 45 kips at point A, drops at a rate of 5 kips/ft. for 12 feet to -15 kips at the end of the distributed load, then goes straight across to point B, and up 15 kips.

The moment curve starts with a parabola going up until the shear curve crosses zero; once the shear is negative, the moment curve drops parabolically until the end of the distributed load. Now the moment curve drops diagonally to point B.



The maximum moment, M_1 , equals the area of the left-hand triangle in the shear diagram. The height of this triangle is 45 kips. We can use the principle of similar triangles to find the base of this

triangle, $x_1 = 12 \text{ ft.} \frac{45 \text{ kips}}{60 \text{ kips}} = 9 \text{ ft.}$, so

$M_1 = \frac{45 \text{ kips} \times 9 \text{ ft.}}{2} = 202.5 \text{ kip}\cdot\text{ft.}$ The base of the right-hand

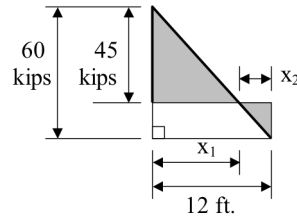
triangle, $x_2 = 12 \text{ ft.} - 9 \text{ ft.} = 3 \text{ ft.}$, so

$M_2 = M_1 - \frac{15 \text{ kips} \times 3 \text{ ft.}}{2} = 180 \text{ kip}\cdot\text{ft.}$ Subtract the area of the

rectangle to find the moment at point B:

$M_3 = M_2 - 15 \text{ kips} \times 12 \text{ ft.} = 0 \text{ kip}\cdot\text{ft.}$

In summary, the value of the moment diagram at a given point equals the area of the shear diagram up to that point. The slope of the moment diagram at a given point equals the value of the shear load at that point.



| Load type | Shear diagram shape | Moment diagram shape |
|-----------------------------------|---------------------------------|---------------------------------|
| Point | Rectangles | Triangles |
| Uniform distributed | Triangles | 1 st order parabolas |
| Nonuniform distributed (triangle) | 1 st order parabolas | 2 nd order parabolas |

Symbols, Terminology, & Typical Units

| | | | |
|---|----------------------------------|------------------|-----------------|
| F | Force | lb., kips | N, kN |
| I | Moment of inertia | in. ⁴ | mm ⁴ |
| L | Length of a beam | ft., in. | m, mm |
| M | Moment | ft.lb., ft.kips | kNm |
| P | Point load | lb., kips | N, kN |
| R | Reaction force | lb., kips | N, kN |
| T | Torque | ft.lb., ft.kips | kNm |
| V | Shear load | lb., kips | N, kN |
| w | Weight per unit length of a beam | lb./ft., kip/ft. | N/m, kN/m |
| W | Weight | lb., kips | N, kN |